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STATISTICAL MODELS FOR THE UTILIZATION PROCESS OF AVIATION RADIO EQUIPMENT

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Abstract. The reliability of aviation equipment is a critical factor that directly influences the efficiency of tasks associated with flight operations. To assess reliability, various indicators are commonly employed, including mean time between failures, mean time between repairs, steady-state availability, availability function, downtime ratio, and utilization factor. However, in modern aviation, the operation of radio equipment often neglects considerations of economic impact, socio-political factors, and a comprehensive analysis of the efficiency of all components within the civil aviation infrastructure. Reliability indicators are typically stochastic in nature, necessitating the development of statistical models, the application of advanced statistical data processing methods, and the enhancement of decision-making technologies, including those leveraging artificial intelligence. External influences, operational conditions, degradation of electrical components, and instability in both autonomous and external power supplies often result in nonstationary trends across the range of parameters being monitored. These dynamic changes highlight the need for advancements in traditional data processing methods, particularly in areas such as dataset formation, classification, evaluation, and forecasting. This article focuses on the development of statistical models for the downtime ratio and utilization factor, specifically addressing scenarios characterized by nonstationary trends in diagnostic parameters.

Keywords: data processing, operation system, aviation radio equipment, nonstationarity, downtime ratio, utilization factor.

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Notations

Variables and functions

$\vec{A}$ – the set of algorithms;
A_c – the steady-state availability;
$\overrightarrow{Con}$ – the vector of external conditions;
Ef – the efficiency indicator;
$E(K_d)$ – the expected value of the downtime ratio;
$E(K_u)$ – the expected value of the utilization factor;
$E(T_0)$ – the expected value of the mean time between failures;
$E(T_M)$ – the expected value of the average duration of maintenance;
$E(T_r)$ – the expected value of the mean time between repairs;
$f(t_i)$, $f(t_{ri})$, $f(t_{di})$, $f(K_d)$, and $f(K_u)$ – the probability density function of operating time between failures, time of repair, downtime duration, downtime ratio, and utilization factor;
$f(\tau)$ – the probability density function of time moment of deterioration occurrence;
K_d – the downtime ratio;
K_u – the utilization factor;
k – the failure number, which corresponds to the beginning of degradation;
$\vec{M}$ – the vector of models;
m_t, m_{tr}, and m_{td} – the expected values of normal distribution of operating time between failures, time of repair, and downtime duration;
N – the sample size;
n – the number of repetition procedures during simulation;
$\overrightarrow{OS}$ – the vector characterizing organizational structure of the operation system;
$\vec{R}$ – the vector of requirements and restrictions;
$\vec{S}$ – the vector of states;
$\vec{T}$ – the vector of trajectories of state changes;
T_0 – the mean time between failures;
T_M – the mean time to perform maintenance;
T_r – the mean time between repairs;
t_i, t_{ri}, and t_{di} – operating time between failures, time of repair, and downtime duration;
α – technical condition deterioration parameter;

$\gamma(\cdot)$ – the partial gamma function;
$\mu(T_0)$, $\mu(T_M)$, $\mu(T_r)$, $\mu(K_d)$, and $\mu(K_u)$ – the variance of the mean time between failures, time of repair, downtime duration, downtime ratio, and utilization factor;
λ_0 – the failure rate;
λ_r – the rates of repair procedure;
λ_d – the rates of downtime;
$\vec{\rho}$ – the vector of requirements of consumers and customers;
σ_t, σ_{tr}, and σ_{td} – the variances of normal distribution of operating time between failures, time of repair, and downtime duration;
$\vec{\theta}$ – the vector of operation system components;
τ – the time moment of deterioration occurrence.

Abbreviations

OS – operation system;
PDF – probability density function.

1. Introduction

Civil aviation is an important component of the transport infrastructure. The main task of civil aviation is to carry passengers and cargo with a given level of high safety and regularity (Cusick et al., 2017). In certain situations, it can offer services and perform tasks that are beyond the capabilities of other transport systems.

Civil aviation includes aircraft, runways, airports, airfields, aviation technical bases, state regulatory bodies, educational and research institutes, aviation equipment manufacturing plants, and airlines (Maleviti, 2023). In general, civil aviation is a system of systems where the listed functional components operate in coordination and interact seamlessly with one another (Kearns, 2021).

The description of its construction and the peculiarities of structural components interaction is based on the principles of system and process approaches (Meijer, 2021). While designing civil aviation systems, it is necessary to apply new ideas, concepts, approaches, and principles, in particular artificial intelligence technologies in all their diversity using knowledge bases, databases, machine and deep learning tools, and others (Andre, 2019).

One of the civil aviation systems is the system for radio technical support of flights. This system includes:

1) devices of communication, navigation, surveillance, security and fire alarm systems, access control and video surveillance, maintenance control, and aviation telecommunication systems;

2) personnel of the operating enterprise, repair organizations, aviation security services, air traffic control centers, and others;

3) regulatory and technical documentation that establishes rules, regulations, restrictions, features of technological operations, and others;

4) means of technological equipment (buildings, structures, workplaces, stands, control and measuring equipment, instruments, spare parts, etc.);

5) main and redundant power supply sources (Stacey, 2008).

From a point of view of hierarchical principles, the listed components form the operation system of aviation radio equipment, where the corresponding equipment is the main element (Solomentsev et al., 2016).

From the point of view of a systemic approach, while creating and modernizing operation systems (OS), it is necessary to take into account the standards, normative documents, and recommendations of international organizations such as ICAO, Eurocontrol, EASA, and FAA (Humphreys, 2023; Ostroumov et al., 2025). This necessitates addressing challenges related to the analysis of domestic regulatory frameworks, the implementation of provisions established by international aviation organisations for international flights, and the harmonisation of global best practices.

Thus, since civil aviation includes a variety of systems, this leads to an increase in the level of uncertainty of the state of certain elements, which arises as a result of either uncontrolled random events or incorrect actions related to the human factor. Uncertainty leads to increased risks of air navigation services, possible unnecessary costs, a decrease in operational efficiency, and the attractiveness of civil aviation in general (Smerichevskyi et al., 2023). Approaches according to which all processes should be performed under controlled conditions based on a set of relevant parameters monitoring can be considered as preventive and protective mechanisms (Karadžić et al., 2012). This corresponds to the generally accepted provisions of the process approach and quality control, each process has an input, resource support, control influences, and output, which are quantitatively characterized by certain parameters (Mitra, 2016).

The main component in the system of radio technical support of flight is equipment. The quality of equipment operation is characterized by tactical and technical characteristics, operational characteristics, and economic characteristics. Tactical and technical characteristics reveal the capabilities of the equipment from the point of view of its functional purpose (Palicot, 2013). Operational characteristics include reliability indicators and diagnostic parameters (Dhillon, 2006). Economic characteristics include tariff rates for the implementation of operational processes, costs for spare parts, power supply, personnel remuneration, and others (Poberezhna, 2017).

Equipment reliability indicators are considered within the framework of four areas, namely serviceability, maintainability, reparability, and durability (Modarres & Groth, 2023).

The main reliability indicators are:

1. Mean time between failures;
2. Mean time between repairs;
3. Steady-state availability;
4. Downtime ratio;
5. Utilization factor;
6. Availability function;

7. Failure rate.
8. Probability of failure-free operation (Rausand, 2004; Smith, 2021).

The values of reliability indicators are determined at the design stage of the equipment and are specified during operation. The assessment of reliability indicators is carried out by collecting, processing, and using statistical data (Prokopenko, 2021). A review of the literature reveals significant focus on failure rate models and steady-state availability (Nakagawa, 2005; Solomentsev et al., 2013). However, the development of models for the downtime ratio and utilization factor has not been adequately addressed.

At the same time, the utilization factor is particularly significant when evaluating the effectiveness of equipment in fulfilling its intended functional purpose (Raza & Ulansky, 2021).

In other words, it can be considered as the coefficient of equipment efficiency (Goncharenko, 2018). Indeed, the equipment must perform useful functions in terms of establishing radio communications, radar surveillance, determining navigation parameters, etc. It means that the use of the equipment must be economically attractive and at the same time the equipment must function reliably and stably (Zaliskyi et al., 2024). Therefore, in the data processing system, attention must be paid to the utilization factor, in particular for assessment procedures, decision-making, and forecasting possible events (Solomentsev et al., 2019).

The aviation radio equipment operation practice shows that the value of estimates of the utilization factor in the general case is a random variable (Ulansky & Raza, 2024). In this case, the definition of statistical models of probability density function and point characteristics (expected value, variance, and standard deviation) of the utilization factor are not considered in the literature. This leads to the fact that it is impossible to determine the limit values, precautionary threshold levels, complicates the process of developing a forecasting model, and others (Gališanskis, 2004).

Academic literature indicates that during the operation and storage of equipment, deviations from stationary conditions can occur. These deviations often result in a deterioration of the equipment's technical condition, leading to complex patterns and trends in monitoring data (Okoro et al., 2022; Tachinina et al., 2021). This phenomenon, commonly referred to as a changepoint problem, is an objective process. It also poses a significant challenge in the calculation and development of statistical models for equipment reliability indicators.

Building on the considerations outlined above, a generalised mathematical formulation of the problem is proposed. Assume that we have a certain organizational structure of the operation system, where the equipment is utilized for its functional purpose, and activities such as maintenance, repair, and resource extension are carried out. This system can be characterized by the vector $\overrightarrow{OS}$. The operation system includes a vector of components $\vec{\theta}$, for which the corresponding states $\vec{S}$, trajectories of state changes $\vec{T}$ are inherent, that is, it is possible to write the operation system in the form of a functional dependence of the form $\overrightarrow{OS}\left(\vec{\theta} \mid \vec{S}, \vec{T}\right)$. During the operation of the radio equipment, stochastic processes of failure occurrence, repair durations and downtime durations, as well as processes of changepoint occurrence are characterized by the vector of models $\vec{M}$. Monitoring data are collected, processed, and used for decision-making based on a set of algorithms $\vec{A}$. All processes are performed under certain operating conditions, which are characterized by the vector of external conditions $\overrightarrow{Con}$. Certain requirements and restrictions can be imposed on the operation processes, which are characterized by the vector $\vec{R}$. In this case, the requirements of consumers and customers can be separately put forward using the vector $\vec{\rho}$. Therefore, the efficiency indicator in the form of the reliability level is a functional dependence of the type:

$$Ef = \varphi\left(\overrightarrow{OS}\left(\vec{\theta} \mid \vec{S}, \vec{T}, \vec{\rho}\right), \vec{M}, \vec{A}, \overrightarrow{Con} \mid \vec{R}\right). \quad (1)$$

During design, efficiency should be ensured at the maximum level or not below a certain threshold (Zhukov et al., 2024). During solution of the problem, it is essential to define a criteria operator that establishes the conditions for improvement, specifying the aspects to be optimised and the target levels to be achieved.

Therefore, the purpose of this paper is to analyse statistical models of reliability indicators for aviation radio equipment, specifically focusing on the utilization factor and downtime ratio. This analysis considers various statistical models of operating times, repair times, and downtime durations, as well as the conditions under which changepoints occur.

2. A generalised methodology for identifying statistical models of the downtime ratio and utilization factor

The downtime ratio is a comprehensive indicator of reliability. It shows the probability that the object will be in a faulty condition at a time remote from the start of operation. From the point of view of the OS, the downtime ratio shows how busy the schedule is for using the equipment for its functional purpose. This coefficient is defined as the ratio of the mean time between repairs T_r to the sum of the mean time between repairs and the mean time between failures T_0:

$$K_d = \frac{T_r}{T_r + T_0}. \quad (2)$$

Equation (2) shows that downtime ratio will be in the range from 0 to 1. If mean time between failures T_0 tends to zero, then downtime ratio will tend to one. If mean time between failures tends to infinity, then downtime ratio will tend to zero.

The downtime ratio is a complement to the steady-state availability A_c. It can also be defined as $K_d = 1 - A_c$. The steady-state availability will be in the range from zero to one.

The utilization factor is also a comprehensive indicator of reliability. It is a performance characteristic that takes into account more factors than the steady-state availability, i.e. all downtime that characterizes the non-use of the equipment for its functional purpose. In the general case, the utilization factor is the ratio of the time the equipment is used for its functional purpose to the total time the equipment is operated.

In the classical sense, the utilization factor can be defined as the ratio of the mean time between failures to the sum of the mean time between failures (T_0), the mean time between repairs (T_r), and the mean time to perform maintenance T_M:

$$K_u = \frac{T_0}{T_0 + T_r + T_M}. \quad (3)$$

The mean time to perform maintenance can be determined based on the statistical data on maintenance performance on corresponding type of radio equipment. In this case, it is necessary to find the average value. This data can be supplemented using recommendations of designers and manufacturers, requirements of normative documentation for continuity of services providing, and taking into account economic features of maintenance process that limit operating costs.

The utilization factor characterizes the reliability of the equipment at certain stages of observation. If the equipment is not used for its intended functional purpose for objective reasons of a higher hierarchical level, then there is no need to calculate it.

We will assume that during operation, statistical data on the operating time t_i, repair times t_{ri} and downtime durations t_{di} are collected in the corresponding separate datasets. In this case, the volumes of the datasets are the same and equal to N, i.e. $i \in [1; N]$. Each value in the datasets are independent random variables that are distributed according to one distribution law for each dataset in the absence of degradation and according to several different laws in the event of degradation.

The degradation (deterioration of the technical condition of the equipment) is associated with the presence of cycles and stages of observation, characterized by nonstationary changes in the diagnostic parameters and reliability indicators. Accordingly, at each stage of observation, there are different intervals of quasi-stationarity, at each of which the data are described by different probability density functions or their characteristics.

Nonparametric methods of estimation and statistical classification can be applied to the collected data. On this basis, probability density functions (PDFs) of the operating time t_i, repair times t_{ri} and downtime durations t_{di} can be obtained, which is denoted by $f(t_i)$, $f(t_{ri})$ and $f(t_{di})$, respectively.

First, the methodology for finding a statistical model of the downtime ratio in the absence of degradation is considered. In this case, the PDF as the statistical model is considered.

The first stage of the methodology involves finding the PDF of the mean time between failures and the mean time between repairs, which is denoted by $f(T_0)$ and $f(T_r)$, respectively. In this case:

$$T_0 = \frac{1}{N}\sum_{i=1}^{N} t_i. \quad (4)$$

$$T_r = \frac{1}{N}\sum_{i=1}^{N} t_{ri}. \quad (5)$$

To find the PDF of the sum of random variables, the standard method of functional transformations of random variables is used.

In this case, it is possible to write:

$$\int_0^{\infty} f(T_0)\,dT_0 = \int_0^{\infty}\cdots\int_0^{\infty} f(t_1)f(t_2)\ldots f(t_N)\,dt_1 dt_2\ldots dt_N. \quad (6)$$

Then:

$$f(T_0) = \int_0^{\infty}\cdots\int_0^{\infty} f(t_1)f(t_2)\ldots f(t_N)\left|\frac{dt_1}{dT_0}\right| dt_2\ldots dt_N, \quad (7)$$

where $\left|\frac{dt_1}{dT_0}\right|$ is Jacobian of the transformation. In this case:

$$t_1 = NT_0 - \sum_{i=2}^{N} t_i. \quad (8)$$

So:

$$\left|\frac{dt_1}{dT_0}\right| = N. \quad (9)$$

Then:

$$f(T_0) = N\int_0^{\infty}\cdots\int_0^{\infty} f(t_1)\Big|_{t_1 = NT_0 - \sum_{i=2}^{N} t_i} f(t_2)\ldots f(t_N)\,dt_2\ldots dt_N. \quad (10)$$

Similar procedures can be used for identifying the PDF $f(T_r)$ of the mean time between repairs.

The next step in finding the downtime ratio model involves applying the functional transformation method to the system of two random variables presented in Equation (2). In this case:

$$\int_0^{\infty} f(K_d)\,dK_d = \int_0^{\infty}\int_0^{\infty} f(T_0, T_r)\,dT_0 dT_r. \quad (11)$$

It is assumed that the random variables T_0 and T_r are independent, which simplifies the solution of the design problems. According to Equation (2), the inverse function is found, which in this case will be the following:

$$T_r = \frac{K_d}{1 - K_d} T_0. \quad (12)$$

Derivative of the inverse function:

$$\frac{dT_r}{dK_d} = \frac{T_0}{(1 - K_d)^2}. \quad (13)$$

So, we get an equation for the PDF of the downtime ratio:

$$f\left(K_d\right)=\int_0^{\infty}\frac{T_0}{\left(1-K_d\right)^2}f\left(T_r\right)\Big|_{T_r=\frac{K_dT_0}{1-K_d}}f\left(T_0\right)dT_0. \quad (14)$$

Equation (14) is generalized. During the research, it is necessary to find a specific equation for $f\left(K_d\right)$ for given distributions of the initial sets in the case of degradation absence.

Next, the method of finding a statistical model of the utilization factor in the case of degradation absence is considered.

The first stage of the method involves finding the PDFs for the mean time between failures, the mean time between repairs and the average duration of downtime (maintenance), which is denoted by $f(T_0)$, $f(T_r)$ and $f(T_d)$ respectively. In this case:

$$T_d=\frac{1}{N}\sum_{i=1}^{N}t_{di}. \quad (15)$$

To find these PDFs, the same approach is used as described in Equations (6)–(10).

The next stage is to determine the PDF of the utilization factor based on the method of functional transformations of the system of three random variables. In this case:

$$\int_0^{\infty}f\left(K_u\right)dK_u=\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}f\left(T_0,T_r,T_d\right)dT_0dT_rdT_d. \quad (16)$$

It is assumed that the random variables T_0, T_r and T_d are independent, which simplifies the solution of design problems. According to Equation (3), the inverse function is found, which in this case will be the following:

$$T_0=\frac{K_u}{1-K_u}\left(T_r+T_d\right). \quad (17)$$

Derivative of the inverse function:

$$\frac{dT_0}{dK_u}=\frac{T_r+T_d}{\left(1-K_u\right)^2}. \quad (18)$$

Therefore, an Equation is obtained for the PDF of the utilization factor:

$$f\left(K_u\right)=\int_0^{\infty}\int_0^{\infty}\frac{T_r+T_d}{\left(1-K_u\right)^2}f\left(T_0\right)\Big|_{T_0=\frac{K_d\left(T_r+T_d\right)}{1-K_d}}f\left(T_r\right)f\left(T_d\right)dT_rdT_d. \quad (19)$$

Equation (19) is generalized. During the research, it is necessary to find a specific equation for $f(K_u)$ for given distributions of the initial sets in the case of degradation absence.

When performing engineering calculations, known equations are used that allow to obtain approximate estimates of statistical characteristics – expected values and variances (Hahn & Shapiro, 1994). According to this approach, the expected value for the downtime ratio and the utilization factor is determined as follows:

$$E\left(K_d\right)=\frac{E\left(T_r\right)}{E\left(T_r\right)+E\left(T_0\right)}; \quad (20)$$

$$E\left(K_u\right)=\frac{E\left(T_0\right)}{E\left(T_0\right)+E\left(T_r\right)+E\left(T_M\right)}, \quad (21)$$

where $E(T_r)$ is an expected value of the mean time between repairs; $E(T_0)$ is an expected value of the mean time between failures; $E(T_M)$ is an expected value of the average duration of technical maintenance.

The variances for the downtime ratio and utilization factor are determined as follows:

$$\mu\left(K_d\right)=\left(\frac{\partial K_d}{\partial T_r}\right)^2\Bigg|_{\substack{E(T_0)\\E(T_r)}}\mu\left(T_r\right)+\left(\frac{\partial K_d}{\partial T_0}\right)^2\Bigg|_{\substack{E(T_0)\\E(T_r)}}\mu\left(T_0\right); \quad (22)$$

$$\mu\left(K_u\right)=\left(\frac{\partial K_u}{\partial T_r}\right)^2\Bigg|_{\substack{E(T_0)\\E(T_r)\\E(T_M)}}\mu\left(T_r\right)+\left(\frac{\partial K_u}{\partial T_0}\right)^2\Bigg|_{\substack{E(T_0)\\E(T_r)\\E(T_M)}}\mu\left(T_0\right)+\left(\frac{\partial K_u}{\partial T_M}\right)^2\Bigg|_{\substack{E(T_0)\\E(T_r)\\E(T_M)}}\mu\left(T_M\right), \quad (23)$$

where $\mu(T_r)$ is a variance of the mean time between repairs, $\mu(T_0)$ is a variance of the mean time between failures, $\mu(T_M)$ is a variance of the average duration of maintenance.

In Equations (22) and (23), the derivatives are defined at the points of expected values of the estimates T_r, T_0, and T_M. In this case, the derivatives:

$$\frac{\partial K_d}{\partial T_r}=\frac{\left(T_r+T_0\right)-T_r}{\left(T_r+T_0\right)^2}=\frac{T_0}{\left(T_r+T_0\right)^2}. \quad (24)$$

$$\frac{\partial K_d}{\partial T_0}=\frac{-T_r}{\left(T_r+T_0\right)^2}. \quad (25)$$

$$\frac{\partial K_u}{\partial T_r}=\frac{-T_0}{\left(T_r+T_0+T_M\right)^2}. \quad (26)$$

$$\frac{\partial K_u}{\partial T_M}=\frac{-T_0}{\left(T_r+T_0+T_M\right)^2}. \quad (27)$$

$$\frac{\partial K_u}{\partial T_0}=\frac{\left(T_r+T_0+T_M\right)-T_0}{\left(T_r+T_0+T_M\right)^2}=\frac{T_r+T_M}{\left(T_r+T_0+T_M\right)^2}. \quad (28)$$

Substituting Equations (24)–(28) into Equations (22) and (23), it is obtained:

$$\mu\left(K_d\right)=\frac{E^2\left(T_0\right)\mu\left(T_r\right)+E^2\left(T_r\right)\mu\left(T_0\right)}{\left(E\left(T_0\right)+E\left(T_r\right)\right)^4}; \quad (29)$$

$$\mu\left(K_u\right)=\frac{\left(E\left(T_r\right)+E\left(T_M\right)\right)^2\mu\left(T_0\right)+E^2\left(T_0\right)\left(\mu\left(T_r\right)+\mu\left(T_M\right)\right)}{\left(E\left(T_0\right)+E\left(T_r\right)+E\left(T_M\right)\right)^4}. \quad (30)$$

The given approximate formulas for the expected values and variances are based on the assumption that we have a Gaussian probability distribution for the downtime ratio and utilization factor. Knowing these parameters, the threshold levels of the corresponding coefficients and their confidence probabilities can be found when forming corrective and preventive actions (Gallo et al., 2023). In general, in operational units, approximate estimates of the downtime ratio and utilization factor can be used to manage the technical condition of radio equipment.

Consider the features of the methodology for determining the PDFs of the downtime ratio and utilization factor in the case of degradation. In addition to information about the type of PDF at individual stages of quasi-stationarity, it is necessary to know the boundaries of these quasi-stationarity intervals, as well as the PDFs of these boundaries.

Let the model of technical condition deterioration is linear. Then, before the degradation, the times of failure occurrence are stationary, and after the degradation, the failure rate begins to grow linearly, i.e., the stationarity of the process is violated. The degradation model is schematically shown in Figure 1.

This model of degradation means that during normal operation the failure rate is constant and equals to λ_0. The change in the technical condition of the aviation radio equipment occurs at a random time τ, which is described by the probability density function $f(\tau)$. After the changepoint occurrence, the failure rate increases according to a linear law characterized by a constant inclination angle α. That is, after the change in the technical condition, the failure rate is defined as $\lambda_1 = \lambda_0 + \alpha\left(t - \tau\right)$.

Analysis of the specifics of the failure occurrence allows to conclude that it is possible to simplify the consideration of reliability analysis processes when presenting them in a discrete form. In this case, the failure rate after the changepoint will be $\lambda_1\left(i\right) = \lambda_0 + \alpha\sum_{j=k}^{i} t_j$, where i is the current number of failures, k is the failure number corresponding to the moment of the changepoint occurrence τ.

In this paper, it was assumed that the type of probability density function of operating time between failures

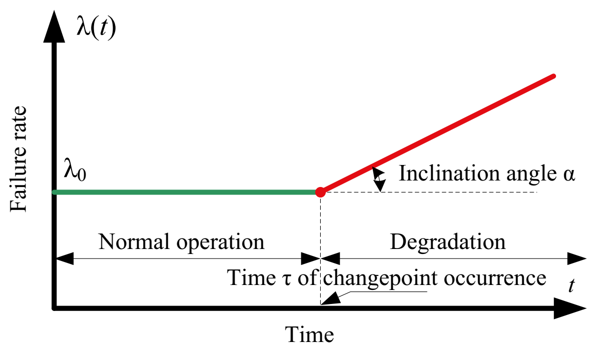

Figure 1. The changepoint model

does not change in the event of degradation, but its parameters (expected value, variance, etc.) change.

The law of change of expected values will be defined by the Equation (31):

$$T_0\left(i\right) = \begin{cases} \dfrac{1}{\lambda_0}, if\ i < k, \\ \dfrac{1}{\lambda_0 + \alpha\sum_{j=k}^{i} t_j}, if\ i \geq k. \end{cases} \quad (31)$$

Such a change of the expected value leads to the fact that the process of failure occurrence after changepoint becomes dependent. Therefore, the PDF will be conditional and will be determined by all previous failures after the changepoint.

To determine the exact analytical equations of the PDFs of the downtime ratio and the utilization factor, Equations (2) and (3) will be written taking into account Equations (4), (5) and (15):

$$K_d = \frac{\sum_{i=1}^{N} t_{ri}}{\sum_{i=1}^{N} t_{ri} + \sum_{i=1}^{N} t_i}; \quad (32)$$

$$K_u = \frac{\sum_{i=1}^{N} t_i}{\sum_{i=1}^{N} t_i + \sum_{i=1}^{N} t_{ri} + \sum_{i=1}^{N} t_{di}}. \quad (33)$$

In this case, the sum is represented of the times of failure occurrence as:

$$\sum_{i=1}^{N} t_i = \sum_{i=1}^{k-1} t_i + \sum_{i=k}^{N} t_i. \quad (34)$$

The PDF of the first term of Equation (34) can be determined using a method similar to (6)–(10). In this case, expression (10) can be represented as:

$$f\left(\Sigma_1\right) = \int_0^\infty \ldots \int_0^\infty f\left(\Sigma_1 - t_2\right) f\left(t_2 - t_3\right) \ldots \\ f\left(t_{k-2} - t_{k-1}\right) f\left(t_{k-1}\right) dt_2 \ldots dt_{k-1}. \quad (35)$$

The PDF of the second term of Equation (34) can be determined similarly only taking into account that each PDF in the sum has the different expected value, then

$$f\left(\Sigma_2\right) = \int_0^\infty \ldots \int_0^\infty f_k\left(\Sigma_2 - t_{k+1}\right) \ldots \\ f_{N-1}\left(t_{N-1} - t_N\right) f_N\left(t_N\right) dt_{k+1} \ldots dt_N. \quad (36)$$

In the Equation (36), the index for each PDF corresponds to the expected value, i.e. the k-th density f_k has the expected value $T_0(k)$, the N-th density f_N has the expected value $T_0(N)$.

To find the PDF of the sum (34), the following Equation is used:

$$f\left(\Sigma_t\right) = \int_0^\infty f\left(\Sigma_1\right)\Big|_{\Sigma_1 = \Sigma_{t_i} - \Sigma_2} f\left(\Sigma_2\right) d\Sigma_2. \quad (37)$$

The PDF of the sums included in Equations (32) and (33) are determined similarly to Equation (35), i.e.:

$$f\left(\Sigma_{t_r}\right)=\int_0^\infty\ldots\int_0^\infty f\left(\Sigma_{t_r}-t_{r(2)}\right)\ldots$$
$$f\left(t_{r(k-2)}-t_{r(k-1)}\right)f\left(t_{r(k-1)}\right)dt_{r(2)}\ldots dt_{r(k-1)}. \quad (38)$$

$$f\left(\Sigma_{t_d}\right)=\int_0^\infty\ldots\int_0^\infty f\left(\Sigma_{t_d}-t_{r(2)}\right)\ldots$$
$$f\left(t_{d(k-2)}-t_{d(k-1)}\right)f\left(t_{d(k-1)}\right)dt_{d(2)}\ldots dt_{d(k-1)}. \quad (39)$$

The next step is to find the PDF of the sum of the random variables in the denominator of Equations (32) and (33). In the case of the downtime ratio, the sum of two random variables is used, i.e.:

$$f\left(\Sigma_{D_1}\right)=\int_0^\infty f\left(\Sigma_t\right)\Big|_{\Sigma_t=\Sigma_{D_1}-\Sigma_{t_r}} f\left(\Sigma_{t_r}\right)d\Sigma_{t_r}, \quad (40)$$

and in the case of the utilization factor, the sum of three random variables is used, i.e.:

$$f\left(\Sigma_{D_2}\right)=$$
$$\int_0^\infty\int_0^\infty f\left(\Sigma_{t_i}\right)\Big|_{\Sigma_{t_i}=\Sigma_{D_2}-\Sigma_{t_d}} f\left(\Sigma_{t_d}\right)\Big|_{\Sigma_{t_d}=\Sigma_{t_d}-\Sigma_{t_r}} f\left(\Sigma_{t_r}\right)d\Sigma_{t_d}d\Sigma_{t_r}. \quad (41)$$

The performed functional transformations will reduce Equations (32) and (33) to the form of two random variables ratio, i.e. final formulas for the PDFs of the downtime ratio and utilization factor can be obtained:

$$f\left(K_d\right)=\int_0^\infty \Sigma_{D_1} f\left(\Sigma_{t_r}\right)\Big|_{\Sigma_{t_r}=K_d\Sigma_{D_1}} f\left(\Sigma_{D_1}\right)d\Sigma_{D_1}. \quad (42)$$

$$f\left(K_u\right)=\int_0^\infty \Sigma_{D_2} f\left(\Sigma_t\right)\Big|_{\Sigma_t=K_d\Sigma_{D_2}} f\left(\Sigma_{D_2}\right)d\Sigma_{D_2}. \quad (43)$$

As it can be seen, the method of finding the PDFs of the downtime ratio and utilization factor is significantly complicated in the case of deterioration of the technical condition of aviation radio equipment. However, in engineering calculations it is also possible to evaluate only the moments of random variables, limiting ourselves to the expected value and variance. Thus, the equations for the expected values will take the form:

$$E\left(K_d\right)=\frac{NE\left(t_r\right)}{NE\left(t_r\right)+\sum_{i=1}^{N}E(t_i)}. \quad (44)$$

$$E\left(K_u\right)=\frac{\sum_{i=1}^{N}E(t_i)}{\sum_{i=1}^{N}E(t_i)+NE\left(t_r\right)+NE\left(t_d\right)}. \quad (45)$$

The variances of statistical estimates will be determined by the Equations:

$$\mu\left(K_d\right)=\left(\frac{\partial K_d}{\partial T_r}\right)^2\Bigg|_{\substack{E(t_i)\\E(T_r)}}\mu\left(T_r\right)+\sum_{i=1}^{N}\left(\frac{\partial K_d}{\partial t_i}\right)^2\Bigg|_{\substack{E(t_i)\\E(T_r)}}\mu\left(t_i\right). \quad (46)$$

$$\mu\left(K_u\right)=\left(\frac{\partial K_u}{\partial T_r}\right)^2\Bigg|_{\substack{E(t_i)\\E(T_r)\\E(T_M)}}\mu\left(T_r\right)+\left(\frac{\partial K_u}{\partial T_M}\right)^2\Bigg|_{\substack{E(t_i)\\E(T_r)\\E(T_M)}}$$
$$\mu\left(T_M\right)+\sum_{i=1}^{N}\left(\frac{\partial K_u}{\partial t_i}\right)^2\Bigg|_{\substack{E(t_i)\\E(T_r)\\E(T_M)}}\mu\left(t_i\right). \quad (47)$$

After performing calculations similar to Equations (23)–(28), the final formulas for the variances are obtained:

$$\mu\left(K_d\right)=\frac{\left(\frac{k-1}{N}E\left(T_0\right)+\frac{1}{N}\sum_{i=k}^{N}E\left(t_i\right)\right)^2\mu\left(T_r\right)+E^2\left(T_r\right)\left(\frac{k-1}{N}\mu\left(T_0\right)+\sum_{i=k}^{N}\mu\left(t_i\right)\right)}{\left(\frac{k-1}{N}E\left(T_0\right)+\frac{1}{N}\sum_{i=k}^{N}E\left(t_i\right)+E\left(T_r\right)\right)^4}; \quad (48)$$

$$\mu\left(K_u\right)=\frac{\left(E\left(T_r\right)+E\left(T_M\right)\right)^2\left(\frac{k-1}{N}\mu\left(T_0\right)+\sum_{i=k}^{N}\mu\left(t_i\right)\right)+\left(\frac{k-1}{N}E\left(T_0\right)+\frac{1}{N}\sum_{i=k}^{N}E\left(t_i\right)\right)^2\left(\mu\left(T_r\right)+\mu\left(T_M\right)\right)}{\left(\frac{k-1}{N}E\left(T_0\right)+\frac{1}{N}\sum_{i=k}^{N}E\left(t_i\right)+E\left(T_r\right)\right)^4}. \quad (49)$$

As it can be seen, Equations (48) and (49) have become significantly more complicated in the form compared to equations (29) and (30). In general, the use of approximate formulas (44), (45), (48), and (49) significantly simplifies the analysis of statistical characteristics of reliability indicators in the case of deterioration of the technical condition of aviation radio equipment compared to the calculation of the PDFs using Equations (31)–(43).

3. Analysis of reliability indicator models for different priori distributions of operating times, times of repair, and downtime durations

This section considers examples of calculations of the PDFs of the downtime ratio and utilization factor in the case of known a priori distributions of operating times, times of repair, and downtime durations during normal operating of aviation radio equipment.

Three options of the primary data distributions were analyzed:

1. Exponential distributions of operating times, times of repair, and downtime durations with given intensities.
2. Normal distributions of operating times, times of repair, and downtime durations with given expected values and variances.
3. Constant values of times of repair and downtime durations and arbitrary distributions of operating times.

The variant with exponential distributions is the most common in the theory of reliability of technical systems. Such a distribution occurs in the case of the simplest failure flow, which is characterized by the appearance of only one failure in a small observation interval, the absence of dependence between failures, and a constant value of the failure rate. In this case, the PDF of operating times, times of repairs, and downtime durations are as follows:

$$f(t) = e^{-\lambda_0 t}, t \geq 0, \tag{50}$$

$$f(t_r) = e^{-\lambda_r t_r}, t_r \geq 0, \tag{51}$$

$$f(t_d) = e^{-\lambda_d t_d}, t_d \geq 0, \tag{52}$$

where λ_r and λ_d are rates of repair procedure and downtime.

Using the method (6)–(10), it is obtained that the corresponding estimates of the mean values of the PDFs will be distributed according to the gamma distribution of the following form:

$$f(T_0) = \frac{(N\lambda_0)^N (T_0)^{N-1} e^{-N\lambda_0 T_0}}{(N-1)!}, T_0 \geq 0. \tag{53}$$

The formulas for estimating the average duration of repair procedures and downtime will be similar to Equation (53), but with its own rate parameter.

Further, using Equations (14) and (19), the PDFs of the downtime ratio and utilization factor are obtained:

$$f(K_d) = \frac{(2N-1)!(\lambda_0\lambda_r)^N (K_d)^{N-1}(1-K_d)^{N-1}}{(N-1)!(N-1)!(\lambda_r K_d + \lambda_0(1-K_d))^{2N}}. \tag{54}$$

$$f(K_u) = \frac{(3N-1)! N^{3N} (\lambda_0\lambda_r\lambda_d)^N}{(N-1)!(N-1)!(N-1)!} \int_0^{1-K_u} \frac{(K_u)^{N-1} x^{N-1}(1-K_u-x)^{N-1}}{(N\lambda_0 K_u + N\lambda_r x + N\lambda_d(1-x-K_u))^{3N}} dx. \tag{55}$$

If $N = 1$, we will get

$$f(K_d) = \frac{\lambda_0\lambda_r}{(\lambda_r K_d + \lambda_0(1-K_d))^2}; \tag{56}$$

$$f(K_u) = \frac{\lambda_0\lambda_r\lambda_d}{2} \int_0^{1-K_u} \frac{1}{(\lambda_0 K_u + \lambda_r x + \lambda_d(1-x-K_u))^3} dx. \tag{57}$$

Normally distributed random variables are the most common in nature. Such distributions are also used in reliability theory. In this case, the PDF of the operating time between failures looks as follows:

$$f(t) = \frac{1}{\sigma_t\sqrt{2\pi}} e^{\frac{-(t-m_t)^2}{2\sigma_t^2}}, t \geq 0, m_t \gg 3\sigma_t, \tag{58}$$

where m_t and σ_t are expected value and standard deviation of distribution correspondently.

Then the mean of the distribution will also be described by a normal PDF:

$$f(T_0) = \frac{\sqrt{N}}{\sigma_t\sqrt{2\pi}} e^{\frac{-N(T_0-m_t)^2}{2\sigma_t^2}}. \tag{59}$$

Equation similar to Equations (58) and (59) can be obtained for times of repair and downtime durations. In this case, it is assumed that these PDFs depend on the parameters m_{tr}, σ_{tr} and m_{td}, σ_{td}.

Further, using Equations (14) and (19), PDFs for downtime ratio and utilization factor are obtained:

$$f(K_d) = \frac{N}{2\pi\sigma_{tr}\sqrt{\sigma_t^2 + \sigma_{tr}^2}} \int_0^{\infty} x e^{-\frac{N(K_d x - m_{tr})^2}{2\sigma_{tr}^2} - \frac{N(x-m_t-m_{tr})^2}{2(\sigma_t^2+\sigma_{tr}^2)}} dx; \tag{60}$$

$$f(K_u) = \frac{N}{2\pi\sigma_t\sqrt{\sigma_t^2 + \sigma_{tr}^2 + \sigma_{td}^2}} \int_0^{\infty} x e^{-\frac{N(K_d x - m_t)^2}{2\sigma_t^2} - \frac{N(x-m_t-m_{tr}-m_{td})^2}{2(\sigma_t^2+\sigma_{tr}^2+\sigma_{td}^2)}} dx. \tag{61}$$

The integrals in Equations (60) and (61) cannot be calculated analytically, therefore, numerical methods and computer simulation methods are used to solve them.

Consider the case of constant values of the times of repair and downtime durations and arbitrary distributions of operating times between failures. In this case, the methodology for determining the reliability indicators will be significantly simplified, since it will use the theory of transformation of a single random variable.

Let T_r = *const*. Then, according to Equation (2), the inverse function was found, which in this case will be the following:

$$T_0 = T_r\left(\frac{1}{K_d} - 1\right). \tag{62}$$

Derivative of the inverse function:

$$\frac{dT_0}{dK_d} = -\frac{T_r}{(K_d)^2}. \tag{63}$$

Therefore, an Equation is obtained for the PDF of the downtime ratio:

$$f(K_d) = \frac{T_r}{(K_d)^2} f(T_0)\Big|_{T_0 = T_r\left(\frac{1}{K_d}-1\right)}. \tag{64}$$

For the case of the utilization factor, we have $T_r + T_d = const$. Then the inverse function and its derivative can be found by Equations (17) and (18). Therefore, the Equation for the PDF of the utilization factor is obtained:

$$f(K_u) = \frac{T_r + T_d}{(1-K_u)^2} f(T_0)\Big|_{T_0 = \frac{K_u}{1-K_u}(T_r+T_d)}. \tag{65}$$

The considered case is much simpler in mathematical terms, since Equations (64) and (65) do not require the calculation of integrals.

This study also considers an example of calculating the PDF of the downtime ratio and utilization factor in the case of known priori distributions of operating times between failures, times of repair and downtime durations while occurring deterioration of the technical condition of aviation radio equipment. The specified analysis for the exponential distribution law is performed, since it is the most general in the theory of reliability.

The calculation using the method corresponding to Equations (35)–(43) is performed.

Until the moment of changepoint, the failure rate is constant, therefore the PDF of the sum of operating times will be determined by Erlang's distribution:

$$f\left(\Sigma_1\right)=\frac{\left(\lambda_0\right)^{k-1}\left(\Sigma_1\right)^{k-2}e^{-\lambda_0\Sigma_1}}{\left(k-2\right)!}. \quad (66)$$

After the changepoint, the failure rate increases linearly. Then in this case, the PDF of the sum of the operating times will take the form:

$$f\left(\Sigma_2\right)=\sum_{i=1}^{N-k+1}\left(\prod_{j\neq i}\frac{\lambda_j}{\lambda_j-\lambda_i}\right)\lambda_i e^{-\lambda_i\Sigma_2}. \quad (67)$$

According to Equation (37), we find the PDF of the sum of two random variables with distributions (66) and (67):

$$f\left(\Sigma_t\right)=\sum_{i=1}^{N-k+1}\left(\prod_{j\neq i}\frac{\lambda_j}{\lambda_j-\lambda_i}\right)\lambda_i\frac{\left(\lambda_0\right)^{k-1}e^{-\lambda_i\Sigma_t}}{\left(k-2\right)!}\int_0^{\Sigma_t}x^{k-2}e^{-\left(\lambda_0+\lambda_i\right)x}dx. \quad (68)$$

In Equation (68) the integral is a partial gamma function and can be denoted as $\int_0^{\Sigma_t}x^{k-2}e^{-\left(\lambda_0+\lambda_i\right)x}dx=\gamma\left(k-1,\left(\lambda_0+\lambda_i\right)x\right)$.

Using Equations (38) and (39) the PDF is obtained of the sums of the times of repair and downtimes, which will be distributed according to Erlang's law:

$$f\left(\Sigma_r\right)=\frac{\left(\lambda_r\right)^{N}\left(\Sigma_r\right)^{N-1}e^{-\lambda_r\Sigma_r}}{\left(N-1\right)!}; \quad (69)$$

$$f\left(\Sigma_d\right)=\frac{\left(\lambda_d\right)^{N}\left(\Sigma_d\right)^{N-1}e^{-\lambda_d\Sigma_d}}{\left(N-1\right)!}. \quad (70)$$

Next, we find the PDF of the sum of random variables in the denominator of Equations (32) and (33). In the case of the downtime ratio, we have the sum of two random variables with distributions (68) and (69). Then we obtain:

$$f\left(\Sigma_{D_1}\right)=\sum_{i=1}^{N-k+1}\left(\prod_{j\neq i}\frac{\lambda_j}{\lambda_j-\lambda_i}\right)\lambda_i\frac{\left(\lambda_0\right)^{k-1}\left(\lambda_r\right)^{N}}{\left(k-2\right)!\left(N-1\right)!} \quad (71)$$
$$\int_0^{\Sigma_{D_1}}\gamma\left(k-1,\left(\lambda+\lambda_i\right)x\right)\left(\Sigma_{D_1}-x\right)^{N-1}e^{-\lambda_i x-\lambda_r\left(\Sigma_{D_1}-x\right)}dx.$$

Using Equation (42) and taking into account Equations (69) and (71), it is obtained:

$$f\left(K_d\right)=\frac{\left(\lambda_r\right)^{N}\left(K_d\right)^{N-1}}{\left(N-1\right)!} \quad (72)$$
$$\int_0^{\infty}\left(\Sigma_{D_1}\right)^{N-2}e^{-\lambda_r K_d\Sigma_{D_1}}f\left(\Sigma_{D_1}\right)d\Sigma_{D_1}.$$

In the case of the utilization factor, we have the sum of three random variables with distributions (68), (69) and (70). Then it is obtained:

$$f\left(\Sigma_{D_2}\right)=\frac{\left(\lambda_0\right)^{k-1}\left(\lambda_r\right)^{N}\left(\lambda_d\right)^{N}\left(\Sigma_{D_2}\right)^{2N-2}}{\left(k-2\right)!\left(N-1\right)!\left(N-1\right)!} \quad (73)$$
$$e^{-\left(\lambda_0+\lambda_r+\lambda_d\right)\Sigma_{D_2}}\sum_{i=1}^{N-k+1}\left(\prod_{j\neq i}\frac{\lambda_j}{\lambda_j-\lambda_i}\right)\lambda_i.$$

Using Equation (43) and taking into account Equations (68) and (73), it is obtained:

$$f\left(K_u\right)=\sum_{i=1}^{N-k+1}\left(\prod_{j\neq i}\frac{\lambda_j}{\lambda_j-\lambda_i}\right)\lambda_i\frac{\left(\lambda_0\right)^{k-1}}{\left(k-2\right)!} \quad (74)$$
$$\int_0^{\infty}\Sigma_{D_2}e^{-\lambda_i K_d\Sigma_{D_2}}\gamma\left(k-1,\left(\lambda+\lambda_i\right)K_d\Sigma_{D_2}\right)f\left(\Sigma_{D_2}\right)d\Sigma_{D_2}.$$

4. Results and discussions

This section is devoted to simulation and calculation of statistical characteristics of estimates of downtime ratio and utilization factor.

The simulation will be based on the following initial data:

- failure rate before changepoint $\lambda_0=0.0001\,\text{hours}^{-1}$;
- recovery rate $\lambda_r=2\,\text{hours}^{-1}$;
- downtime rate $\lambda_d=0.05\,\text{hours}^{-1}$;
- distribution type for modeled data sets is exponential;
- number of observations N = 16;
- failure number, which corresponds to the beginning of degradation k = 11;
- technical condition deterioration parameter α = 0.5;
- number of repetition procedures n = 10000.

First, a simulation is conducted and the results analysed under the assumption that there is no deterioration in the technical condition of the aviation radio equipment.

At the first stage of simulation, exponentially distributed random variables were formed. Three options of simulation results of the operating time between failures are shown in Figure 2.

Next, the mean time between failures is estimated. The resulting histogram of estimates and the PDF according to Equation (53) are presented in Figure 3. It should be noted that in engineering calculations, the obtained PDF can be approximated by a normal law. This conclusion can be obtained as a consequence of the central limit theorem of

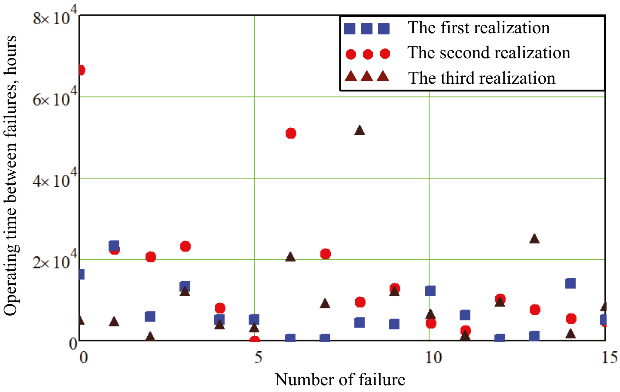

Figure 2. Simulation of operating time between failures (three options)

probability theory, that is, in the case when $N \to \infty$, such a statement is more adequate.

From the point of view of engineering calculations, the normal PDF can simplify the solution of the problem of theoretical calculations due to the existence of a large number of reference and methodological materials, as well as the dependence of the normal distribution on two parameters: the expected value and the standard deviation.

Estimates of the downtime ratio and the utilization factor depending on the number of failures are shown in Figure 4. As it can be seen from the graph, with an increase in the number of failures, the amplitude of fluctuations of the estimates decreases and the estimates become more stationary. In cases when $N \to \infty$, it can be concluded that the estimates of the indicated coefficients become constant. This assumption allows for theoretical research and calculations.

Figure 5 presents a graph illustrating the relationship between the utilization factor estimates and the duration of downtime. It is obvious that long downtimes reduce the level of reliability. In particular, the annual downtime of equipment reduces the utilization factor by approximately 50% for studied initial parameters. As the duration of downtime increases for the given initial parameters, as well as in other scenarios, the utilization factor progressively approaches zero. Introducing a specific threshold level for the utilization factor can address the practical challenge of extending the service life of aviation radio equipment. In practice, the service life determined at the design stage is often not entirely accurate. Manufacturers tend to underestimate this value, as they have a vested

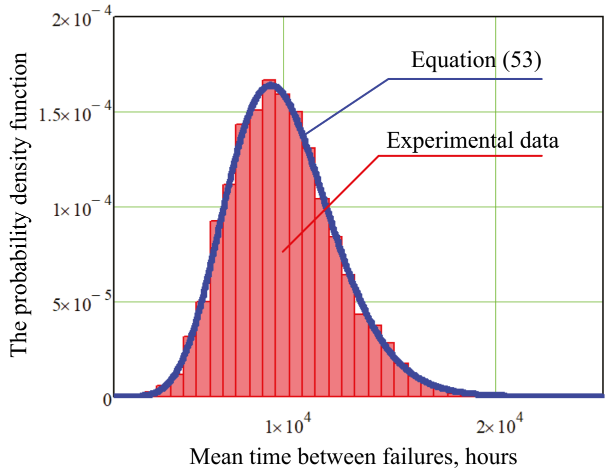

Figure 3. The probability density function and the histogram of mean time between failures estimates

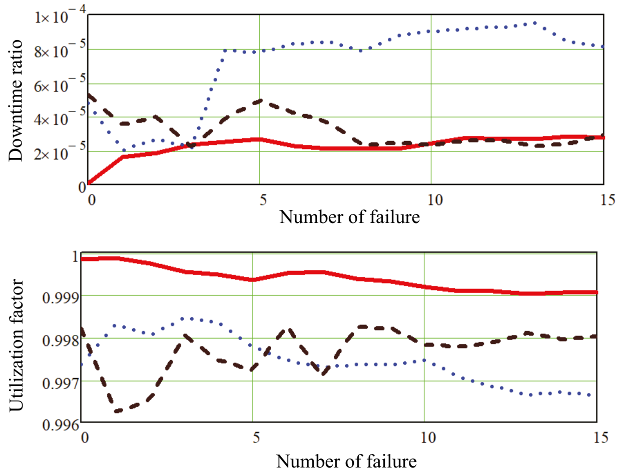

Figure 4. Estimates of downtime ratio and utilization factor depending on the number of failures

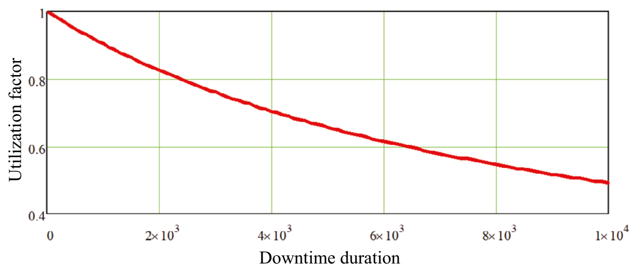

Figure 5. Estimates of utilization factor depending on downtime

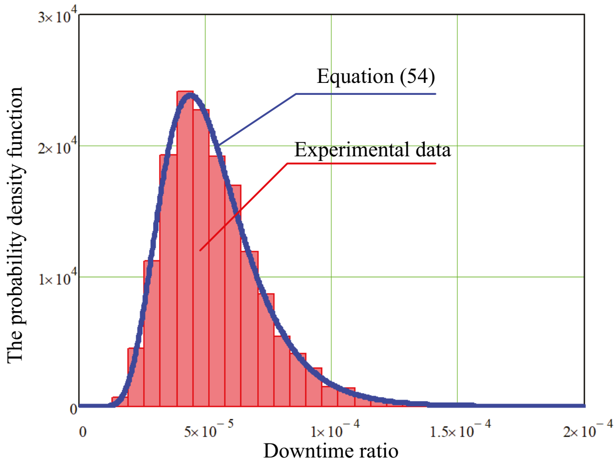

Figure 6. The probability density function and the histogram of downtime ratio estimates

interest in promoting the purchase and replacement of operational equipment.

Next, the estimation of the downtime ratio and utilization factor was performed. The resulting histogram of estimates and the PDFs according to Equations (54) and (55) are presented in Figure 6 and Figure 7.

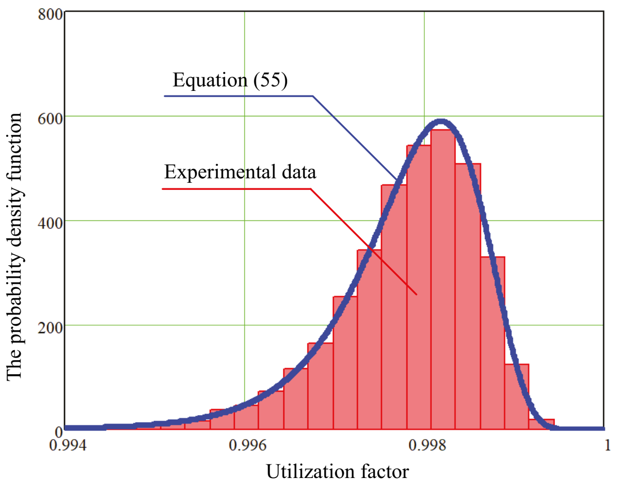

Figure 7. The probability density function and the histogram of utilization factor estimates

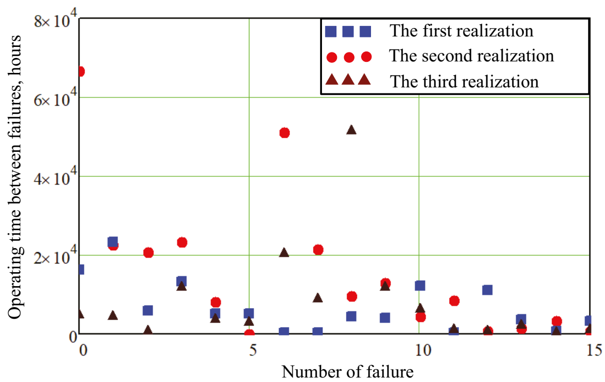

Figure 8. Simulation of operating time between failures (three options)

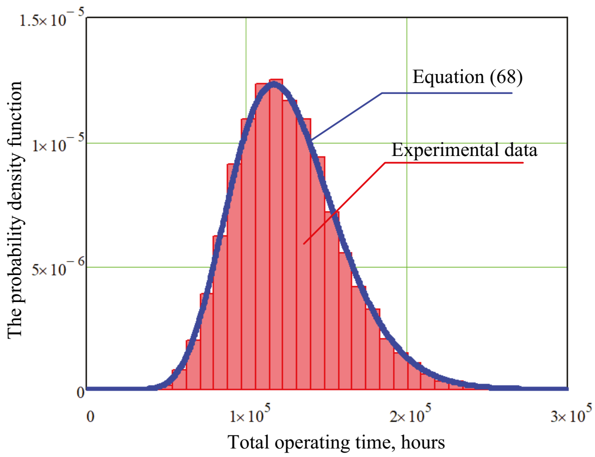

Figure 9. The probability density function and the histogram of total operating time estimates

After that, a simulation and analysis of its results are performed in the case of deterioration of the technical condition of the aviation radio equipment. Three options of simulation results of the operating time between failures are shown in Figure 8. Next, the mean time between failures is estimated. The resulting histogram of estimates and the PDF according to Equation (68) are presented in Figure 9.

The estimates of the utilization factor depending on the downtime in the case of absence and presence of deterioration of the technical condition are shown in Figure 10. The occurrence of deterioration significantly

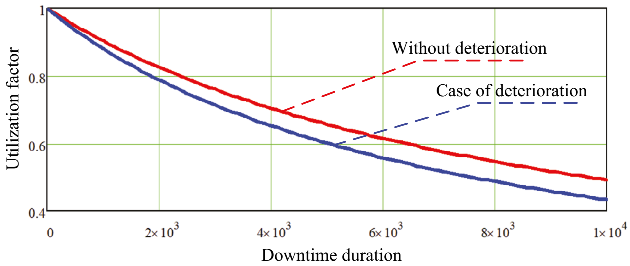

Figure 10. Estimates of utilization factor depending on downtime duration in the cases of absence and presence of deterioration of technical condition

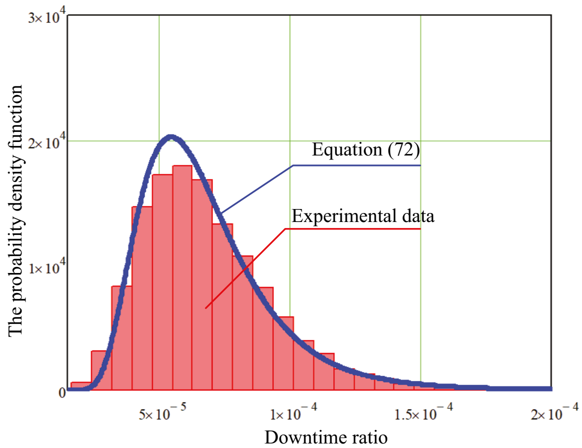

Figure 11. The probability density function and the histogram of downtime ratio

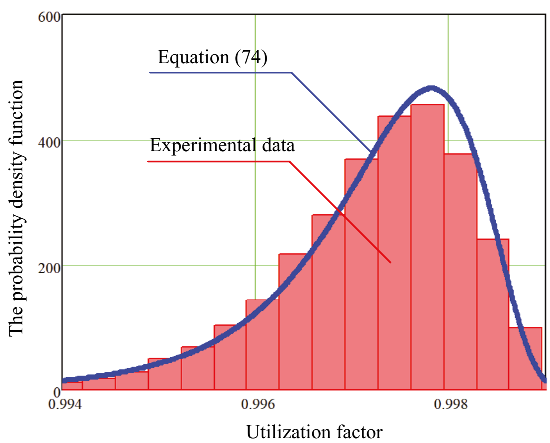

Figure 12. The probability density function and the histogram of utilization factor estimates

Table 1. Point estimates of the downtime ratio

Failure rate	Point estimates based on analytical equations		Estimates based on simulation results		Approximate estimates	
	Mean	Variance	Mean	Variance	Mean	Variance
0.0001	5.311×10^{-5}	3.795×10^{-10}	5.348×10^{-5}	3.855×10^{-10}	5×10^{-5}	4.999×10^{-9}
0.0002	1.050×10^{-4}	1.616×10^{-9}	1.071×10^{-4}	1.567×10^{-9}	9.999×10^{-5}	1.999×10^{-8}
0.0003	1.575×10^{-4}	3.259×10^{-9}	1.595×10^{-4}	3.527×10^{-9}	1.5×10^{-4}	4.497×10^{-8}
0.0004	2.100×10^{-4}	5.792×10^{-9}	2.130×10^{-4}	6.085×10^{-9}	2×10^{-4}	7.994×10^{-8}
0.0005	2.625×10^{-4}	9.049×10^{-9}	2.668×10^{-4}	9.320×10^{-9}	2.499×10^{-4}	1.249×10^{-7}

Table 2. Point estimates of the utilization factor

Failure rate	Point estimates based on analytical equations		Estimates based on simulation results		Approximate estimates	
	Mean	Variance	Mean	Variance	Mean	Variance
0.0001	0.9978	6.396×10^{-7}	0.9978	6.321×10^{-7}	0.9980	8.138×10^{-6}
0.0002	0.9956	2.531×10^{-6}	0.9956	2.559×10^{-6}	0.9959	3.229×10^{-5}
0.0003	0.9935	5.636×10^{-6}	0.9935	5.615×10^{-6}	0.9939	7.206×10^{-5}
0.0004	0.9913	9.914×10^{-6}	0.9914	9.865×10^{-6}	0.9919	1.271×10^{-4}
0.0005	0.9892	1.533×10^{-5}	0.9892	1.458×10^{-5}	0.9899	1.969×10^{-5}

reduces the value of the utilization factor. At the final stage of simulation, the estimation of the downtime ratio and utilization factor was performed. The resulting histogram of estimates and the PDFs according to Equations (72) and (74) are presented in Figure 11 and Figure 12.

Visual analysis of the obtained graphs in Figures 3, 6, 7, 9, 11, and 12 allows to conclude that the analytical correlations coincide with the simulation results. For a more thorough analysis and correct conclusion, the Pearson chi-square test was used. According to this test, for a significance level of 0.05, it was concluded that the data do not contradict the theoretical distribution.

The results of calculating the point estimates of the reliability indicators and their comparison with the exact values and estimates based on the simulation results are given in Tables 1 and 2.

As evidenced by the data in Tables 1 and 2, the approximate formulas used for engineering calculations of point characteristics of distributions provide reasonably accurate estimates of the expected values. For the analysed initial data, the error does not exceed 8%. However, variance estimates obtained from these approximate formulas are overestimated by approximately an order of magnitude. Despite this, the level of accuracy may still be adequate for engineering calculations.

5. Conclusions

The article considers the issue of determining statistical models for the downtime ratio and the utilization factor for equipment that can be used for radio technical support of aircraft flights. These reliability indicators are stochastic. Therefore, the main attention in the article was paid to the definition of analytical equations regarding the most complete characteristic of a random variable – the probability density function. Obtaining these formulas became possible due to the use of the methods of functional transformations of random variables. The veracity and correctness of the obtained mathematical equations were checked by statistical simulation and application of the Pearson chi-square test. According to the results of the calculations, the analytical formulas coincide with the simulation results at the significance level of 0.05. Therefore, the obtained statistical models are reliable.

During the mathematical models building in the analytical form, the influence of changepoint in the trend of the reliability indicator was researched. The analysis showed that the presence of the changepoint significantly complicates analytical calculations. According to the simulation results, it can be seen that the presence of technical condition deterioration leads to fluctuations in the estimates of the downtime ratio and utilization factor, and after the changepoint, the downtime ratio increases, and the utilization factor decreases, after which they both acquire a stable value. The fact that reliability deteriorates in accordance with the values of the studied indicators necessitates the development and application of changepoint detection algorithms.

Exact analytical formulas for PDFs of reliability indicators and their moments are difficult to apply in practice in the relevant aviation enterprises. Therefore, the article proposes approximate formulas for the expected value and variance for estimates of the downtime ratio and the utilization factor. The given example of calculating expected value and variances of estimates for cases of using exact formulas, simulation results and approximate formulas showed that the error in determining the expected value of the estimate does not exceed 8%.

The results of the research can be useful for design organizations, scientific institutions, and aviation enterprises that study and operate radio equipment in the process of initial design and modernization of equipment operation systems.

Future scope

To develop effective preventive and corrective actions within the operating system (OS) of aviation radio equipment, it is recommended to implement procedures for forecasting trends in diagnostic parameters and reliability indicators. This approach is highly advantageous, as proactively addressing potential issues is more cost-effective than incurring unnecessary expenses to mitigate their negative consequences.

Thus, it is essential to explore comprehensive data processing procedures that encompass monitoring, information collection, direct data analysis, and the application of results for forecasting, diagnostics, evaluation, and informed decision-making in management.

Another future research direction involves the development of alternative methodologies for processing non-stationary sets of reliability parameters. This includes the design of algorithms for multi-criteria optimisation, leveraging advanced technologies and artificial intelligence approaches to identify optimal solutions effectively.

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Author contributions

Conceptualization and methodology: MZ, OS and IO; investigation: VI and NK; resources: OS, VI and NK; data curation: OS; writing – original draft preparation: MZ and OS; writing – review and editing: MZ and VI; visualization: IO and NK; validation: MZ; supervision: OV and VI. All authors have read and agreed to the published version of the manuscript.

Disclosure statement

The authors declare no conflict of interest.

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