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ANALYSIS FOR SYMMETRICAL AND ASYMMETRICAL LOADING OF A SINGLE-SPAN COMBINED STRING STEEL STRUCTURE

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Abstract. The article consists of 3 parts. String analyzes with symmetrical loading, combined parametric analyzes of string structure with symmetrical loading and combined string structure analyzes with asymmetrical loading. Through various parameters, the influence of different parameters of the string on its behavior is revealed. The influence of prestressing in the combined string structure is also released. To assess the correctness of the results, an experimental study was carried out in the laboratory, after the structure was designed from steel elements.

Keywords: steel bridge, symmetrical load, asymmetrical load, suspended cable, string, nonlinear analysis, displacements, strain, parametrical analysis.

Introduction

Suspension structures are one of the most rational structures (Gimsing, 1997; Strasky, 2005; Bleicher, 2011; Chen, 2014, Greco et al., 2014). The main advantages of such structures: elegance, low steel consumption. However, the low weight of the structure has its negative aspects. Such structures are sensitive to asymmetric effects, and the smaller the dead weight, the more the structure deforms (Juozapaitis et al., 2006). Such structures can be made more efficient by using string structures (Beivydas, 2022). Such structures do not have initial sag and kinematic displacements. However, such constructions are difficult to design, because their analytical methodologies have not been fully explored.

In order to reduce the shear forces and general displacements, the string can be transformed into a combined structure (Sandovič & Juozapaitis, 2012; Unitsky, 2006). It is additionally supported by a suspended single-span structure (Beivydas, 2019). In any case, in order to eliminate the kinematic displacements of suspended structures, we can combine them with a string structure, which does not have such displacements (Beivydas, 2018).

However, the design of such structures is very complex and difficult. Not enough research has been done on the calculations of such structures. Most of the developed calculation methodologies are either designed for cables (Chen et al., 2014; Juozapaitis & Norkus, 2005) and suspension bridges (Chen et al., 2014) or they are adapted for the calculation of individual elements. However, there have been studies on two-spant suspension systems, where the calculation assumptions and possible methods have been revealed (Sandovič et al., 2011). However, such structures reduce kinematic displacements less effectively, and also, they do not create a straight contour of the upper element, which is also very relevant.

Since the main design criterion for suspended structures is their deformability and stiffness (Kulbach, 1999; Kmet & Kokorudova, 2009; Schlaich et al., 2011), the structure is analyzed by choosing different composite parameters (Beivydas, 2022).

Boundary parameters are selected for string structure analysis, where the inverse problem is solved. The string structure and the combined string structure, which are presented in Sections 1 and 2, are analyzed separately. The results of the experimental study are also presented for the combined string structure.

1. Sring structure analysis

Methods for calculating displacements of a string from a symmetrical loading are already known. The analysis is carried out by choosing the limit vertical displacements, solving the inverse problem, searching for rational string

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parameters. The deflections of the analyzed structure are limited depending on the length of the span, i.e. three alternatives are investigated: $\Delta f_{\text{lim}} = \left(\frac{L}{400}, \frac{L}{250}, \frac{L}{100}\right)$, where L is the span length, equal to 25 and 50 m.

By using the familiar string displacement equation (Beivydas, 2022), we can calculate the required cross-sectional area (Eqs (1)–(2)):

$$\Delta f = \sqrt[3]{\frac{3}{64} \cdot \frac{(p+q)L^4}{EA}}. \quad (1)$$

The required cross-section area is calculated:

$$A = \frac{3}{64} \cdot \frac{(p+q)L^4}{E\Delta f_{\text{lim}}^3}. \quad (2)$$

When the string is pre-tensioned, the methodology below applies:

$$A = \frac{3}{64} \cdot \frac{(p+q)L^4(1-\beta)}{E\Delta f_{\text{lim}}^3}; \quad (3)$$

$$\beta = \frac{T}{H_1}; \quad (4)$$

$$H_1 = T + H; \quad (5)$$

$$H = \sqrt[3]{\frac{1}{24} \cdot \frac{EA \cdot (p+q)^2 L^2}{(1-\beta)}}, \quad (6)$$

where Δf – displacement in the middle of the span; $p + q$ – dead and live loads (2.5 + 7.5 kN/m); E – modulus of elasticity; A – string cross-sectional area; H_1 – tension force, when the string is prestressed; H – tension force, when the string is not prestressed; T – prestressing force; $\Delta_{f\text{lim}}$ – displacement limit.

In Tables 1 and 2, we can see that for both 25 meters and 50 meters spans, the pre-tensioning of the string is of particular importance here. In order to reduce the mass and deformations of the structure, the string must be pre-tensioned (Beivydas, 2022). However, looking at the Table 3, we can notice that the length of the span also has a significant influence on the efficiency of the structure. We can see that by increasing the prestress of the structure when we have a span of 50 meters, the mass of the structure increases much faster than when we have a span of 25 meters.

Depending on the limit displacements, we can draw the following conclusions, in accordance with the criterion of saving materials, the string, when a separate element, should be chosen with a smaller span. Particularly large spans should be avoided when the limit displacements of the string are larger.

In Figure 1 we can see the mass distribution graphically. Although the preload increases, at a larger span, the mass increases strongly. This occurs especially when larger string deformations are allowed.

Table 1. Dependence of the required cross-sectional area of the string on the limit displacements and prestressing force at $L = 25$ m

Prestressing force T, kN	$L/400$	$L/250$	$L/100$
	String cross-sectional area A, m^2		
0	3.75	0.92	0.06
200	3.69	0.89	0.06
600	3.58	0.85	0.05
1000	3.47	0.81	0.04
1200	3.42	0.79	0.04
4000	2.84	0.59	0.02
6000	2.53	0.49	0.02
8000	2.29	0.41	0.01
10000	2.08	0.36	0.01
20000	1.44	0.20	0.01

Table 2. Dependence of string cross-section on ultimate deflection and prestress at $L = 50$ m

Prestressing force T, kN	$L/400$	$L/250$	$L/100$
	String cross-sectional area, m^2		
0	7.5	1.831	0.117
200	7.44	1.808	0.11
600	7.324	1.763	0.107
1000	7.212	1.721	0.101
1200	7.156	1.7	0.098
4000	6.466	1.458	0.071
6000	6.048	1.232	0.06
8000	5.682	1.211	0.051
10000	5.357	1.116	0.045
20000	4.167	0.803	0.028

Table 3. Relative difference in string cross-sections when the spans $L = 50$ and $L = 25$)

Prestressing force T, kN	$L/400$	$L/250$	$L/100$
	The ratio of the cross-sectional areas of the string where span lenght $L = 50$ m and $L = 25$ m		
0	2.0	2.0	2.0
200	2.0	2.0	2.0
600	2.0	2.1	2.1
1000	2.1	2.1	2.5
1200	2.1	2.2	2.5
4000	2.3	2.5	3.6
6000	2.4	2.5	4.0
8000	2.5	3.0	5.1
10000	2.6	3.1	6.0
20000	2.9	4.0	6.6

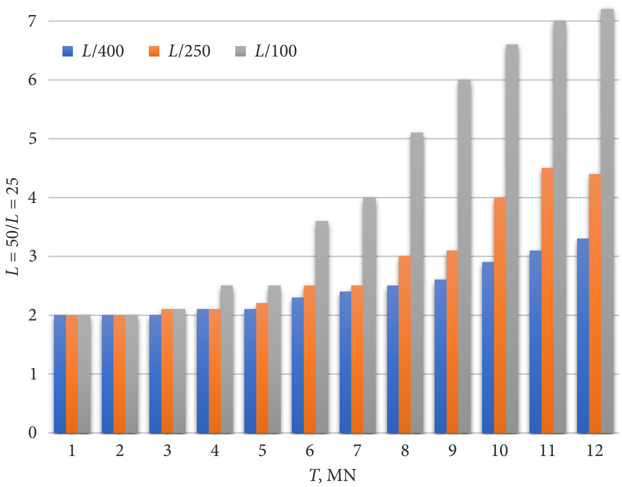

Figure 1. Relative difference in mass at span lenght L = 50 m and L = 25 m

2. The analysis of the combined string structure

The combined structure is a combination of a single-span and string structure. As seen in Figure 2, the string is supported by auxiliary cable using struts. For the analysis of the combined structure, the familiar approximate calculation expressions (Eq. (7)) are used.

From these equations, we can notice what is required for the axial stiffness of the string and cable, depending on all parameters (Beivydas, 2019).

$$A_{cable} \cong \frac{3}{128} \cdot \frac{L^4}{E_{cable} \cdot \Delta f_{Lim} \cdot f_0^2} \cdot \left[\sqrt[3]{\sqrt{\frac{1}{27} \cdot C^3 + \frac{1}{4} \cdot D^2} - \frac{1}{2} \cdot D} + \sqrt[3]{-\sqrt{\frac{1}{27} \cdot C^3 + \frac{1}{4} \cdot D^2} - \frac{1}{2} \cdot D} \right]; \quad (7)$$

$$C = \frac{32768}{9} \cdot \frac{E_{cable}^3 \cdot A_{cable}^3 \cdot f_0^6}{E_{string} \cdot A_{string} \cdot L^8}; \quad (8)$$

$$D = (p+q) \cdot \frac{32768}{9} \cdot \frac{E_{cable}^3 \cdot A_{cable}^3 \cdot f_0^6}{E_{string} \cdot A_{string} \cdot L^8}. \quad (9)$$

Equations are used to calculate the cross-sectional areas of the string and the lower cable at the limit deflections at different axial stiffness ratios of the string and the lower cable. 3 options of the difference in axial stiffness are selected (the ratio of the cross-sectional areas of the string and the cable), see Table 4 and Figure 3. The axial stiffnesses of the cable, the axial stiffness of the string and the total axial stiffness are calculated separately to see the differences between the string and the lower cable.

In order to further reveal the behavior of the combined structure, an experimental study of the structure model was carried out. A 5 meter span combined structure model with 6 struts was designed. Supports are immovable for linear displacements, but movable for angular displacements. The upper and lower elements of the structure are made of 6 mm diameter round steel bar, S355J2 steel grade. Struts and cross beam from pipe CFRHS40×20×2, connecting elements from CFRHS20×20×2, grade S355J2 respectively. The end nodes are attached to the supports with M27 screws, which are used to pre-tension the string (see Figure 4). In order to load the structure with concen-

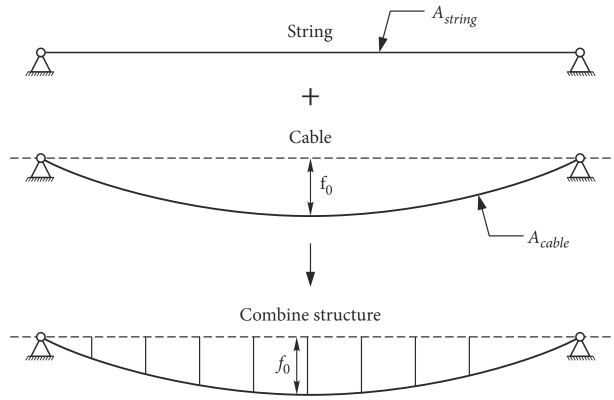

Figure 2. Combined string structure

Table 4. Dependence of cross-sectional areas of combined structure on limit deflections

$\Delta f_{\lim}$	f_0	A, m^2 10^{-4}	n = 3	n = 1	n = 0.5
$L/100$	$L/10$	A_s	22.53	11.38	8.55
		A_c	5.63	5.69	5.70
		A_{tot}	64.71	22.42	11.32
	$L/20$	A_s	86.28	44.84	33.96
		A_c	21.57	22.42	22.64
		A_{tot}	311.79	127.03	67.25
	$L/50$	A_s	415.72	254.06	201.75
		A_c	103.93	127.03	134.5
		A_{tot}	519.86	381.09	336.25
$L/250$	$L/10$	A_s	57.00	28.56	21.44
		A_c	14.25	14.28	14.29
		A_{tot}	169.86	56.98	28.54
	$L/20$	A_s	226.48	113.96	85.61
		A_c	56.62	56.98	57.07
		A_{tot}	1011.18	350.28	176.87
	$L/50$	A_s	1348.24	700.56	530.61
		A_c	337.06	350.28	353.74
		A_{tot}	16.9	5.69	2.851
$L/400$	$L/10$	A_s	68.49	22.86	11.43
		A_c	22.83	22.86	22.87
		A_{tot}	91.32	45.72	34.30
	$L/20$	A_s	273.36	91.35	45.71
		A_c	91.12	91.35	91.41
		A_{tot}	364.48	182.70	137.12
	$L/50$	A_s	1675.68	567.21	284.71
		A_c	558.56	567.21	569.42
		A_{tot}	2234.24	1134.42	854.13

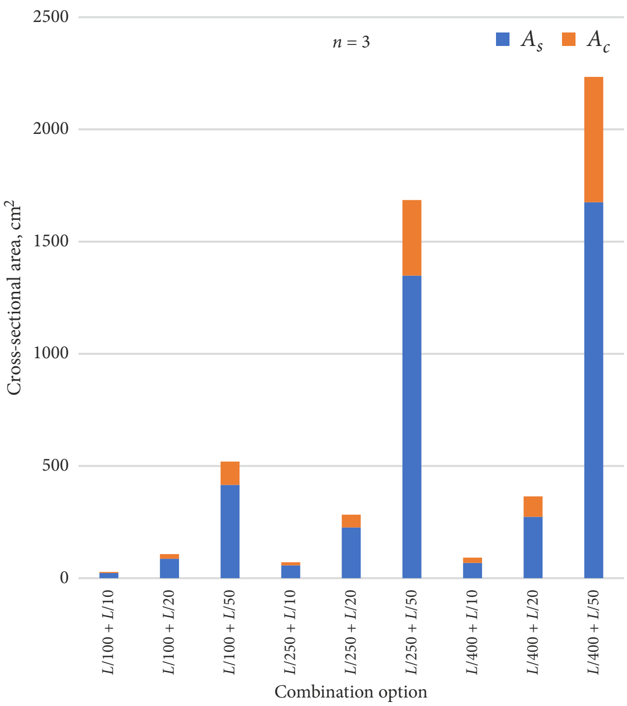

Figure 3. Dependence of the cross-sectional areas of the string and the lower cable on the limit displacements and the initial sag of the lower rope when $n = 3$. Here: n – the ratio of the cross-sectional areas of the string and the lower cable

trated loads at the struts, a wooden deck made of wooden dowels 50×50 mm was installed on the structure, with the help of which the entire load is transferred to the struts. The structural model was loaded with 3 variants of asymmetric load of different sizes (see Table 5), depending on the ratio of live to dead load.

$$\gamma = \frac{q}{p},$$

where γ – the ratio of live to dead load, respectively equal to 1, 2 ir 3, q – live load size, p – dead load size.

The structural model was loaded with steel bars (weights), keeping one row of bars as one load step. The total load transferred to one struts is 0.36 kN for two bars or 0.18 kN per bar to one strut. The pre-stress was installed under dead load. The loading scheme of the model is presented in Figure 5.

In the structure, the displacements of the nodes of the lower and upper bar were measured, saying that the deformations of the struts are insignificant and the upper bar deforms analogously. Deflection gauges were installed on all struts (see Figure 2). Figure 6 shows the displacement graph of the asymmetrically loaded model. We can see that the structure subjected to an asymmetric load deforms in an S-shaped outline. The main reason for such deformation is the resulting kinematic displacements in the lower cable, which lead to the S-shaped deformation of the structure. Comparing experimental and numerical results, we see that the differences in deformations are not large and reach no more than 10 percent. A smaller difference is observed in the unloaded part, which reaches up to 4 percent. In Figure 7, we can see the influence of prestress on kinematic displacements. During the experiment (see Figure 8), the pretension of the string was 2 kN. In the numerical experiment, by increasing the prestress to 10 kN, we can see how much influence it has on the displacements. When the ratio of live to dead load is equal to 3, the prestressing force reduces to 47 percent displacement on the side of the model loaded with asymmetric load, and on the unloaded part – as much as 75 percent lower displacement

Table 5. Variations of loads to struts

Load type	Load to internode, kN	Stage load, kN	Total load of combination, kN
Dead load	0.361	2.524	2.524
Live load, Y1	0.361	1.262	3.786
Live load, Y2	0.361	1.262	5.048
Live load, Y3	0.361	1.262	6.310

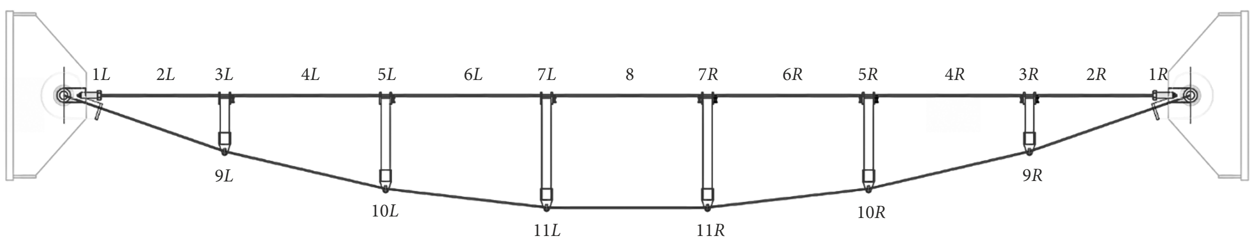

Figure 4. Schematic diagram of the experimental model, where 1, 2, 3… is number of node, L – left side, R – right side

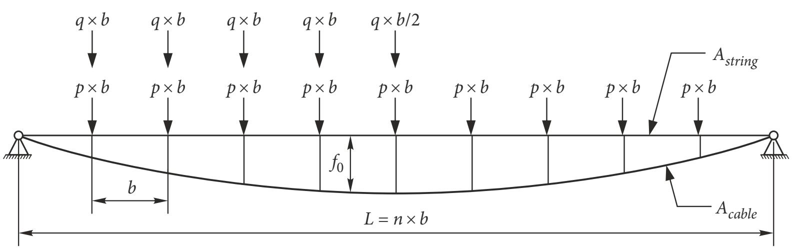

Figure 5. The principle of the model loading scheme, loading the structure with concentrated loads. Here b – the width of the internode; p – dead load Load per linear meter, q – live load Load per linear meter

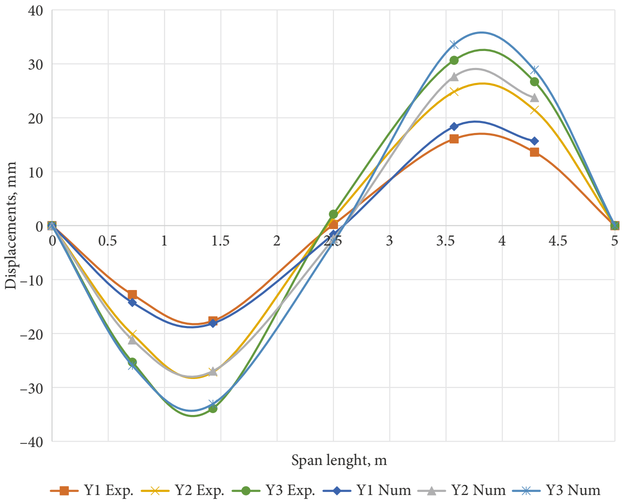

Figure 6. Displacements results of the experimental ant numerical investigation of the combined string structure, where Exp – experimental results, Num – numerical results

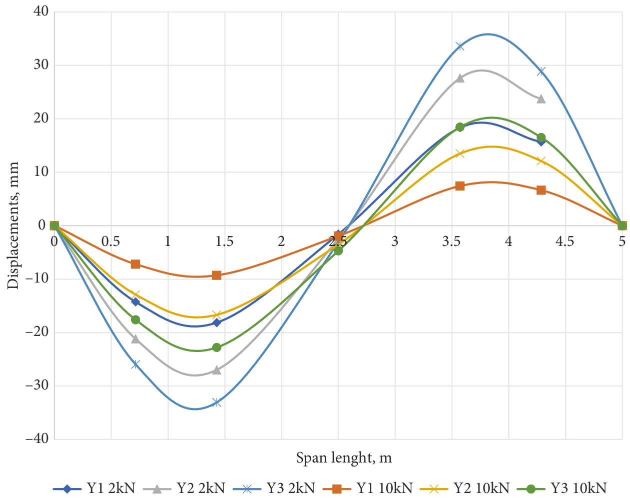

Figure 7. Displacements results of the numerical test of the combined string structure, when string pre-tension is 2 and 10 kN

Figure 8. Photo of the experimental model and the load during the test

than in the case with 2 kN prestressing. It is necessary to mention that such prestressing in the case of symmetrical loading reduces displacements to only 1%. These results reveal the influence that the string has on constraining the kinematic displacements of the lower cable. I.e. although in the case of symmetric loading the string does not have a significant influence, but in the case of asymmetric loading, its influence is essential for the displacements. Since the string has no kinematic displacements, it constrains the kinematic displacements of the lower cable.

Conclusions

1. When composing a single string according to the limit displacements, we can see that according to the material cost criteria, the string as an individual load-bearing element has a relatively high mass and should be used to overlap significantly smaller spans. Particularly large spans should be avoided when the limit displacements of the string are smaller.
2. String should not be used for large spans because its elastic displacements are too large.
3. As for the combined structure, which consists of a string and a suspension cable with initial sag, it is necessary to emphasize that it has quite a few advantages when compared to a string. However, a lower cable with initial sag will always experience kinematic displacements when loaded with an asymmetric load.
4. By combining these structural elements into one whole, the problems of both the string and the suspension cable behavior are partially solved. The string in such a construction significantly constrains the kinematic displacements of the lowee cable.
5. Strings are stabilized by increasing their pre-tension. I.e. it takes over a larger part of the kinematic displacements, as a result of which the total displacements of structure.
6. After testing the experimental model and comparing the results with the numerical ones, when the structure is loaded asymmetrically, up to 10% differences in the unloaded part of stucture and up to 4 percent differences in the loaded part of the structure.

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