



## DAM DEFORMATION MEASUREMENTS WITH GPS

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**Abstract.** The results of deformation measurements are related to direct safety of engineering structures and human life. To avoid a wrong interpretation of displacements, an appropriate deformation monitoring network must be established and the data obtained from deformation monitoring network must be carefully evaluated. Deformation measurements and analysis require the use of very accurate surveying equipment and analysis methods. The Global Positioning System (GPS) meets the requirements stated above and therefore GPS receivers were used for this research. The purpose of this work was to monitor and analyze the deformation at the crest of the Altınkaya dam caused by the water load at different water levels combined with the dam's weight. The secondary goal was to determine whether GPS measurements could meet the accuracy requirements for dam deformation measurements. As working area the Altınkaya dam is selected it is rockfill. In order to monitor and examine the deformation, a monitoring network consisting of 6 reference points and 11 object points was established. Measurements were made 4 times over 2 years using dual frequency GPS receivers with static methods. The measurements were performed and point coordinates have been determined. Then differences were calculated between periods and analyzed by iterative weighted transformation and Least Absolute Sum methods to determine the points stability.

**Keywords:** dam, Global Positioning System, deformation measurements, deformation analysis.

### 1. Introduction

Deformation measurements are one of the most important activities of engineering surveying. In the settlement areas with a dense population engineering buildings as dams, bridges, viaduct and skyscrapers have been built. Thus, the importance of deformation measurements have been increased. These kinds of engineering buildings are exposed to deformation due to different factors of changes as tectonic movements, landslides, changings on ground water level etc. Dams are affected and deformed because of internal and external loads. These loads are not constant and can change over time. Deformation and seepage are functions of these loads. The effects of deformation on rock-filled dams are altogether different from that of concrete dams. However, the deformation of both types is largely characterised as permanent. The weight of a dam and the hydrostatic pressure of reservoir back water yield forces which result in vertical movements of structures. The hydrostatic pressure of a reservoir can also cause a permanent horizontal deformation that is perpendicular to centreline of the dam. Elastic behaviour is smaller in a rock-fill dam (Engineer manual 1994). The electric power in Turkey is to a large extent generated from the water temporarily stored in reservoirs. Deformations of points on dam crest have so far mainly been defined by geodetic measurements. There is a variety of surveying instruments and methods with different capacities and features that are

used in monitoring deformations. Conventional control surveying can be used to monitor the motion of points on a structure relative to stable monuments. Photogrammetric methods are also suitable for dam deformation surveying. Techniques of digital photogrammetry depict a fast, economic and versatile tool for 3-D deformation measuring. Terrestrial laser scanners have recently taken large steps in development and have the potential to become a useful survey tool.

The purpose of this work is to monitor and analyze the deformations at the crest of the Altınkaya dam which were caused by the water load at different levels combined with the dam's weight. A secondary goal was to determine whether GPS measurements could meet the accuracy requirements for dam deformation measurements. Deformation analyses were made using both an Iterative Weighted Similarity Transformation (IWST) and the Least Absolute Sum (LAS) method which does not require previous information about deformation behaviour. Then, the results of the two methods have been compared and discussed.

### 2. WGS 84 Cartesian coordinates transformed to local topocentric coordinate system

The coordinates of GPS measurements is WGS-84 Cartesian coordinate system. Thus, in order to see the real directions of the displacements, all WGS-84 Cartesian

coordinates have been transformed to a local topocentric coordinate system because the obtained directions in this system are incompatible with directions on the physical ground. Thus, Cartesian coordinates in the WGS-84 coordinate system must transform the plane coordinate system. For this process, deformation network must be connected at least at three points of known coordinates in the national coordinate system. This process requires more time and additional process. In this work, Cartesian coordinates in the WGS-84 system are transformed to the local topocentric coordinate system. Because this method is more useful and easier, no additional measurements are required to obtain the local topocentric coordinates.

The origin of local topocentric coordinate system ( $P_i$ ) is at the ground point. Because of this feature the topocentric system ( $u$ ) coordinate axis is taken as the surface normal of ellipsoid passing from ground point ( $P_i$ ) that is selected as the origin point. Positive direction of this axis is selected because it is directed towards the zenith. Axis ( $n$ ) surface normal of reference ellipsoid on the ground point ( $P_i$ ) is taken as the origin point of system and meridian plane including polar point (Fig. 1).

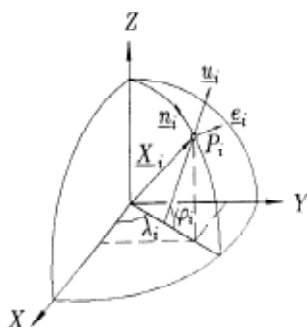


Fig. 1. Local topocentric coordinate system (Wellenhof et al. 1992)

$T$  transformation matrix Eq. (1):

$$T = \begin{bmatrix} -\sin \lambda & \cos \lambda & 0 \\ -\sin \varphi \cos \lambda & -\sin \varphi \sin \lambda & \cos \varphi \\ \cos \varphi \cos \lambda & \cos \varphi \sin \lambda & \sin \varphi \end{bmatrix}. \quad (1)$$

Horizontal plane ( $X_0, Y_0, Z_0$ ) is tangent in the point with Cartesian coordinate.  $e, n, u$  are components in the right-hand system and Cartesian coordinates are transformed relative to Eq. (2):

$$\begin{bmatrix} e_i \\ n_i \\ u_i \end{bmatrix} = T \left[ \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} - \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \right]. \quad (2)$$

Variance-covariance matrix of Local Topocentric Coordinate System is given in Eq. (3):

$$V_{CV_{ENH}} = T \cdot V_{CV_{XYZ}} \cdot T^T, \quad (3)$$

where  $V_{CV_{XYZ}}$  is Variance-covariance matrix of Cartesian coordinate system. This equation is used for transformation to Local Topocentric Coordinate System from GPS coordinates.

### 3. Robust methods for analyzing displacements

In this work, for determining the displacements of deformation monitoring network IWST (Iterative Weighted Similarity Transformation) and LAS robust methods are used. Robust methods are used when there is no previous information about the movement of points within the network (Singh, Setan 1999). IWST, developed by Chen (Chen 1983) in the New Brunswick University, is known as the robust method.

Calculated displacement values could be affected from data selection or from defining two different data while adjusting the measurements taken at two different periods (Chen 1983). Therefore, the weight matrix is obtained iteratively (Singh, Sefan 1999; Chen 1983; Gökalp 1997). Adjusted coordinates of the points  $\hat{x}_1, \hat{x}_2$  in the deformation network and their cofactor (covariance) matrices  $Q_{\hat{x}_1}, Q_{\hat{x}_2}$  are calculated with two separate adjustments. Displacement values ( $d$ ) and the cofactor matrix of  $d$   $Q_d$  are calculated as

$$d = \hat{x}_2 - \hat{x}_1, \quad (4)$$

$$Q_d = Q_{\hat{x}_1} + Q_{\hat{x}_2}. \quad (5)$$

At the beginning, weight matrix ( $W$ ) are accepted as  $W = I$ . This indicates that all the points in the network have the same importance. Therefore solution is like helmert transformation. If only some points are given unit weight and the others zero weight, i.e.  $W = \text{diag} \{1, 0\}$  (Chen et al. 1990).

Then displacement values ( $d$ ) are calculated from Eq. 6.

$$d = S(W)d, \quad (6)$$

here  $S(W)$  shows that  $S$  matrix, calculated with  $W = I$ , can be obtained as

$$S = (I - H(H^TWH)^{-1}H^TW). \quad (7)$$

$H$  matrix for the 3D networks is written.

$$H = \begin{bmatrix} e & 0 & 0 & 0 & z_0 & -y_0 & x_0 \\ 0 & e & 0 & -z_0 & 0 & x_0 & y_0 \\ 0 & 0 & e & y_0 & -x_0 & 0 & z_0 \end{bmatrix}_{3m \times 7}, \quad (8)$$

where  $e^T = (1, \dots, 1)$  and  $z_0, y_0, x_0$  are approximate coordinate vectors with respect to centre of the network.

$$\begin{aligned} X_0^i &= X_i - \frac{1}{m} \sum_1^m X_{i0}, \\ Y_0^i &= Y_i - \frac{1}{m} \sum_1^m Y_{i0}, \\ Z_0^i &= Z_i - \frac{1}{m} \sum_1^m Z_{i0}. \end{aligned} \quad (9)$$

Above,  $z_0^i, y_0^i, x_0^i$  are the  $i^{\text{th}}$  elements of  $z_0, y_0, x_0$  respectively, and  $z_i, y_i, x_i$  are approximate coordinates of point  $P_i$  and  $m$  is the number of the points in the network (Kuang 1996; Öztürk, Şerbetçi 1992; Singh, Setan 2001).

The main difference between LAS and IWST method is in forming the weight matrix (Chen 1983). In the IWST and LAS methods, some points in a reference network

cannot be accepted as stable; in other words, not every point has equal importance. Hence, in the beginning, the weight matrix ( $W$ ) is accepted as  $W = I$ . While datum determine, this indicates that all the points in the network have the same importance. Therefore the solution is similar to the Helmert transformation, if some points are given unit weight and the others a zero weight, i.e.  $W = \text{diag} \{I, 0\}$  (Chen 1983; Chen *et al.* 1990). Hence the weight matrix can be obtained iteratively, as shown below.

The weight matrix for IWST,

$$W_k = \text{Diag} \left\{ \frac{1}{|d_{i(k)}|} \right\}, \quad (10/1)$$

where  $d_{i(k)}$  is the  $i^{\text{th}}$  component of the vector  $d_k$  after  $k^{\text{th}}$  iteration.

The weight matrix for LAS,

$$W_k = \text{Diag} \left\{ \frac{1}{\sqrt{(dx_i^k)^2 + (dy_i^k)^2}} \right\}, \quad (10/2)$$

where  $d_{xi}$  and  $d_{yi}$  are displacements in the direction of  $x$  and  $y$  axis respectively.  $d_{k+1}$  is calculated as

$$d_{k+1} = S(W_k). \quad (11)$$

The iterative procedure continues until the user defines threshold ( $\varepsilon$ ):

$$d_{ik+1} - d_{ik} < \varepsilon. \quad (12)$$

If the difference is greater than  $\varepsilon$ ,  $W_k$  is calculated again using the value obtained from Eq. (11). Then,  $Q_{d(k+1)}$  is calculated:

$$Q_{d(k+1)} = S(W_k)Q_{d1}S^T(W_k). \quad (13)$$

Note that during the iterative procedure, some  $d_{ik}$  may be close to 0. In that case,  $W_k$  becomes very large. This could cause numerical instabilities in calculation of  $W_k$ . Therefore, in order to avoid this problem, when the  $W_k$  is calculated, the tolerance value ( $c$ ) is only added to those displacement components ( $d_i$ ) for LAS method (Eq. 10/1) and also the tolerance value ( $c$ ) is only added to those displacement components ( $d_{xi}$  and  $d_{yi}$ ) for LAS method (Eq. 10/2), where tolerance value "c" is a small constant.

In this procedure, LAS method minimizes the sum of the length of displacement

$$\left( \sum \sqrt{(dx_i)^2 + (dy_i)^2} \Rightarrow \text{minimum} \right),$$

while the IWST method minimizes the total sum of absolute values of the displacement components ( $\sum |d_i| \Rightarrow \text{minimum}$ ) (Singh, Sefan 1999). In order to determine unstable reference and object points in the deformation network, the following procedure could be used. Values of  $c_i$  are determined using calculated parameters in Eqs (11) and (13) for each point as in Eq. (14).

$$d_i^2 / |q_{di} \sigma_0^2| = c_i. \quad (14)$$

Here,  $\sigma_0^2$  is determined as

$$\sigma_0^2 = (df_1 \sigma_{01}^2 + df_2 \sigma_{02}^2) / (df_1 + df_2), \quad (15)$$

where  $df_1$  and  $df_2$  are degrees of freedom used while adjusting the first and second period measurements respectively.  $\sigma_{01}^2$  and  $\sigma_{02}^2$  are variance factors

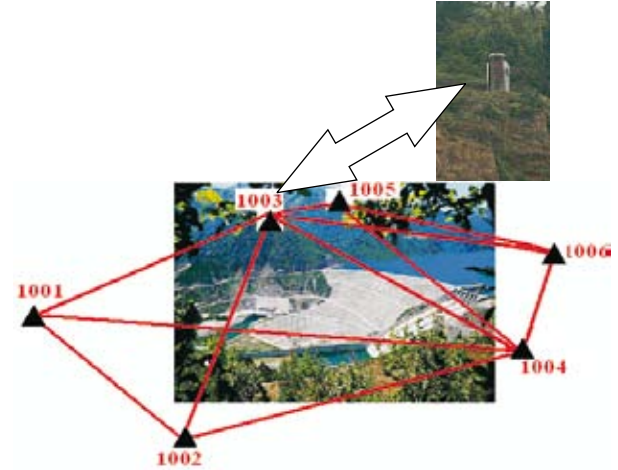
$$c_i < F(1 - \alpha, 1, df). \quad (16)$$

If the above equation is accepted, it may be said that the point is stable, otherwise it is unstable. In Eq. (16),  $\alpha$  is significance level,  $df$  – the degree of freedom calculated from  $df = df_1 + df_2$ .

## 4. Application

### 4.1. Description of Altýnkaya dam

The Altýnkaya dam is located in 35 km southwest from Bafra city of Samsun. It is the second largest embankment dam in Turkey. Fig. 2 shows a view of dam's main structure and also a big part of the reservoir. The dam is built on the Kýzýlýrmak River as rock fill with clay having seeds. Reservoir volume at normal water surface elevation is  $5.76 \times 10^9 \text{ m}^3$ . Annual generation is 1632 GWh. Height of the dam (from the river bed) is 195 m, crest length 634 m. Reservoir area at normal water surface elevation is  $118.31 \text{ km}^2$ . Volume of the dam is  $16 \times 10^6 \text{ m}^3$ . The dam is convex towards the water.



Number of session	1	2	3	4	5
Receiver locations	1003-1005-1006	1003-1004-1006	1003-1004-1002	1003-1001-1002	1001-1004-1005

Fig. 2. Reference network of Altýnkaya dam and measurement plan

### 4.2. Description of Altýnkaya dam geodetic deformation monitoring network

The deformation measurements of the dam involved four measurement campaigns and two separate measurements were made at the Altýnkaya dam: one between reference points and the other within the object points. The deformation measurements related to the reference and object network were made with 3 Ashtech Z surveyor GPS receivers and 700700B\_Mar.III\_L1/L2 GPS antennas. Monitoring network consists of 6 reference stations (1001, 1002, 1003, 1004, 1005, 1006), built as pillars on



Fig. 3. Object network of Altynkaya dam

stable ground surrounding the dam. Fig. 2 shows a view of reference point and the deformation measurement information related to the reference network. For the measurement plan on the reference network, the observation period was selected 45 minutes with a sampling rate of 10 seconds. The satellite elevation mask was selected at  $15^\circ$  in order to avoid multi-path effect and cycle slip error. Measurements at reference points were made on pillars, therefore forced centering was achieved.

In order to monitor and measure possible displacements at the crest, 10 object points were established at the crest, when the dam was built. Since that time only one object point (numbered 0023) was added to the deformation network (Fig. 3). This point was built because of the physical changes that had been observed in the surrounding area. Two different measurement plans were applied to the survey object points. Receiver locations and sessions related to the measurements are given in Fig. 2. In the first plan, two receivers were set over points 1003 and 1004. Then, the third receiver was set over each object point about 30 minutes. In the second plan, a receiver was set over point 0003 during the all observation periods, and the other points were measured using a leapfrog method. The main goal of this measurement plan was to correlate the observations and make loop closures. The object point measurements were taken using tripods with optical plummets and string plumb-bobs used for centring. Before commencing deformation measurements, all the equipment was calibrated. In order to avoid or diminish any equipment errors, the same GPS receivers and antennas were used at the same points in all periods.

#### 4.3. Evaluation of GPS measurements

The measurements were processed with GeoGenius 2000 software. The deformation network showed a maximum value of 0,9 mm horizontally and 1,7 mm vertically for the 4 observational periods. The adjusted coordinates and their covariances were obtained from a free network adjustment. The accuracy of the measurements of the horizontal deformations and the vertical displacements

depends on the accuracy of the instruments used. Thus, the Minimum Norm Quadratic Unbiased Estimation (MINQUE) method was used to determine the accuracy of the baseline measurements instead of taking the values that are given for the GPS receiver and by software manufacturers as above (Gökalp 1991). The accuracy of baselines were calculated 4 mm horizontally and 6 mm vertically by MINQUE (Taşçı, Gökalp 2002). There was little difference between the given and calculated values. Therefore, the default values were used. A 2-dimensional deformation analysis in the Altynkaya dam was made because of a lower accuracy of the Z components with respect to the X and Y components. It was also assumed that the vertical deformation caused by the weight of the dam did not exist. Therefore, only horizontal displacements were determined and analyzed in this study. In order to see the real directions of the displacements, all WGS 84 Cartesian coordinates were transformed to a local topocentric coordinate system. For the transformation, 0003 point has been accepted as the fix and the other points has reduced the datum of this point. Variance-Covariance matrix obtained from GeoGenius GPS software is transformed to Variance-Covariance matrix of Local Topocentric Coordinate system. In the deformation analysis this Variance-Covariance matrix is used and displacements are determined.

#### 4.4. Determination of stable and unstable points in the deformation network using robust methods

In the process determining the movement points in the network with IWST and LAS methods for each period are tested according to  $\alpha = 0,95$ . The first period is taken as the reference period. Therefore, according to reference period, as the periods 1–2, 1–3, 1–4 are formed. Point coordinates E, N and their cofactor matrices  $Q_{x1}$ ,  $Q_{x2}$  were calculated with two separate free network adjustments. The IWST and LAS determined the displacement values (d) with two iterations (appendix 1).

#### 4.5. Interpretation of the results

Water levels were 170.34 m in first period, 167.53 m – in the second period, 164.20 m – in the third period, and 177.23 m – in the final period. In between 3 epochs observed, because of a larger electric production and no rainfall, water level has decreased. The reduction of water level in between the 3 epochs is 6.14 m. This reduction caused the movement of points on the dam's crest. These horizontal movements on the crest could occur in the middle of the dam crest in arch dams was proved by the GPS measurements and deformation analysis methods (Fig. 4).

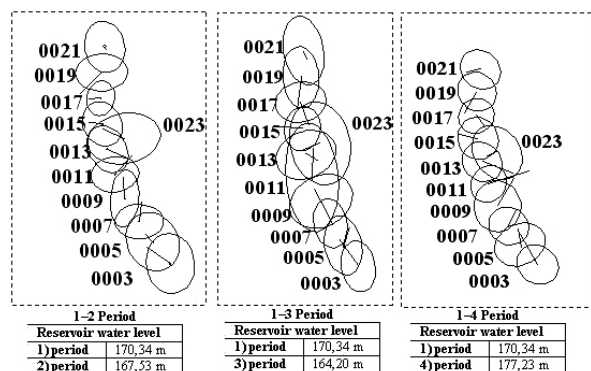


Fig. 4. The deformation ellipses drawing between periods

#### 5. Conclusions

In order to monitor and measure possible displacements at the crest, an extensive deformation network has been set up in the Altýnkaya dam and surrounding region to carry out deformation studies. The study was undertaken to determine whether GPS measurements could reach the accuracy requirements for dam deformation measurements.

GPS is an effective method for static deformation monitoring. There is a direct correlation between fall in the reservoir water level and dam deformation. As the water level of reservoir falls, points on the dam's crest tend to shift towards upstream. In inverse action, the points on the axis of dam tend to shift towards downstream (Fig. 4).

In this study, the most significant movements have been seen in the middle of the dam's crest and at the ends. These phenomena observed from the GPS observation have also been provided by the results of deformation analysis (Fig. 4). The movement of the points on the dam's crest is insufficient to interpretation with respect to water load.

Deformation parameters obtained from the results of two independent techniques indicate a very high correlation. Besides, both methods have the capability of determination of the single point displacements free from the choice of the datum in the reference networks. When comparing the LAS method with the IWST method, the capability of determining the single point displacements of the LAS method is more effective with respect to IWST method. Because, when examining the displacement vectors, obtained by the LAS method, the points with a displacement greater than or equal to 4 mm were accepted as unstable. This coincides with the horizontal accuracy of the observations calculated by MINQUE (Gökalp 1991; Taşçı, Gökalp 2002).

A 2-dimensional deformation analysis was made because of a lower accuracy of the Z component with respect to the X and Y components. Therefore, for further work, in order to determine the movement in the vertical dimension, the use of a precise levelling or another method that has the potential to provide a vertical accuracy equal to or greater than the horizontal accuracy of GPS is recommended.

In conclusion, in regard to a magnitude of the displacements, there is neither threat to the structural and functional security of the dam nor to the lives and property of people living in the vicinity.

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Research interest: Global Positioning System, deformation measurements and analysis.

Appendix 1. Stable and unstable points determined by IWST and LAS

Stable and unstable points determined by IWST and LAS																	
Between 1–2 periods				Between 1–3 periods				Between 1–4 periods									
IWST		LAS		IWST		LAS		IWST		LAS							
dN	dE	dN	dE	dN	dE	dN	dE	dN	dE	dN	dE	dN	dE				
mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm				
1001	-4,9	1,0	-4,8	1,6	stable	IWST	stable	LAS	unstable	IWST	unstable	LAS	unstable	IWST	stable	LAS	unstable
1002	0,0	4,8	0,3	5,5	unstable	IWST	unstable	LAS	unstable	IWST	unstable	LAS	unstable	IWST	stable	LAS	stable
1003	0,1	0,0	1,1	0,9	stable	IWST	stable	LAS	stable	IWST	stable	LAS	unstable	IWST	stable	LAS	stable
1004	-2,3	0,0	-2,1	1,5	stable	IWST	stable	LAS	stable	IWST	stable	LAS	stable	IWST	stable	LAS	stable
1005	-3,5	0,0	-3,3	0,4	stable	IWST	stable	LAS	stable	IWST	stable	LAS	stable	IWST	stable	LAS	stable
1006	-6,2	2,6	-5,9	3,2	unstable	IWST	unstable	LAS	unstable	IWST	unstable	LAS	unstable	IWST	stable	LAS	stable
3	3,9	-4,9	4,3	-4,3	unstable	IWST	unstable	LAS	unstable	IWST	unstable	LAS	unstable	IWST	stable	LAS	unstable
5	1,8	-1,2	2,2	-0,6	stable	IWST	stable	LAS	stable	IWST	stable	LAS	unstable	IWST	stable	LAS	unstable
7	4,6	0,0	4,9	1,0	unstable	IWST	unstable	LAS	unstable	IWST	unstable	LAS	unstable	IWST	stable	LAS	unstable
9	4,9	0,0	5,2	0,4	stable	IWST	stable	LAS	unstable	IWST	stable	LAS	stable	IWST	stable	LAS	unstable
11	4,4	3,6	4,7	4,2	unstable	IWST	unstable	LAS	unstable	IWST	stable	LAS	unstable	IWST	stable	LAS	unstable
13	-2,4	4,7	-2,1	5,3	unstable	IWST	unstable	LAS	unstable	IWST	stable	LAS	unstable	IWST	stable	LAS	unstable
15	0,0	-2,7	0,6	-2,2	stable	IWST	stable	LAS	stable	IWST	unstable	LAS	unstable	IWST	stable	LAS	unstable
17	0,0	-2,5	0,3	-1,9	stable	IWST	stable	LAS	stable	IWST	unstable	LAS	unstable	IWST	stable	LAS	unstable
19	-4,8	-4,3	-4,5	-3,8	unstable	IWST	unstable	LAS	unstable	IWST	unstable	LAS	unstable	IWST	stable	LAS	unstable
21	0,0	0,0	-0,3	0,9	stable	IWST	stable	LAS	stable	IWST	stable	LAS	stable	IWST	stable	LAS	stable
23	2,1	-4,5	2,4	-3,9	stable	IWST	unstable	LAS	unstable	IWST	stable	LAS	stable	IWST	stable	LAS	stable

Reference points

Object points