GEODESY AND CARTOGRAPHY
ISSN 2029-6991 print / ISSN 2029-7009 online
2011 Volume 37(1): 15–21
doi: 10.3846/13921541.2011.558339

Taylor & Francis
Taylor & Francis Group

UDK 528.21

DETERMINATION AND EVALUATION OF THE ESTONIAN FITTED GEOID MODEL EST-GEOID2003

Harli Jürgenson[1], Kristina Türk[2], Jüri Randjärv[3]

Department of Geomatics, Estonian University of Life Sciences (ELS),
Kreutzwaldi 5, Tartu, Estonia
E-mail: [1]harli.jyrgenson@emu.ee (corresponding author)

Received 1 Dec. 2010; accepted 20 Dec. 2010

Abstract. This paper focuses on issues related to the calculation of a high-precision fitted geoid model on Estonian territory. Model Est-Geoid2003 have been used in Estonia several years in geodesy and other applications. New data from precise levelling, new global models and terrestrial gravity data give plenty of possibilities for updates and accuracy evaluation.

The model is based on a gravimetric geoid. From the gravimetric data gathered, a gravimetric geoid for Estonia was calculated as an approximately 3-km net using the FFT method. After including the new gravimetric data gathered, the gravimetric geoid no longer had any significant tilt relative to the height anomalies derived from GPS-levelling points. The standard deviation between the points was 2.7 cm.

The surface of the calculated gravimetric geoid was fitted by high-precision GPS-levelling points. As a result, a height transformation model was determined to reflect the differences between the normal heights of BK77 and the ellipsoidal heights of EUREF-EST97 on Estonian territory. The model was originally called Est-Geoid2003 and is part of the official national geodetic system in Estonia. The model is updated and evaluated here using precise GPS-levelling points obtained from different measurement campaigns.

In 2008–2010 the preliminary results from the latest precise levelling sessions became available, leading to a significant increase in the number of precise GPS-levelling points. Both networks are part of the Estonian integrated geodetic network. Using very precise levelling connections from new levelling lines, normal heights of several RGP points were calculated additionally. Misclosure of 300 km polygons are less than 2–3 mm normally. Ealier all precisely levelled RGP points were included into fitting points. Now many new points are available for fitting and independent evaluation. However, the use of several benchmarks for the same RGP point sometimes results in a 1–2 cm difference in normal height. This reveals problems with the stability of older wall benchmarks, which are widely used in Estonia. Even we recognized, that 0.5 cm fitted geoid model is not achievable using wall benchmarks. New evaluation of the model Est-Geoid2003 is introduced in the light of preliminary data from new precise levelling. Model accuracy is recognised about 1.2 cm as rms.

Keywords: local geoid modelling, fitted geoid model, precise levelling.

1. Introduction

This research was started under PhD studies until 2003 and has continued thereafter. There are also other attempts concerning Baltic geoid (Ellmann 2004) but current paper is dedicated to developments in model Est-Geoid2003. The calculations for geoid determination have been performed with the software package Gravsoft (Tscherning *et al.* 1992). The availability of new satellite-based global models and new gravity data has given continuous opportunities for gravimetric geoid determination. After the high-precision geoid was derived, it was fitted according to high-precision GPSlevelling data. This model was named Est-Geoid2003, and it has been adopted as an official reference model in Estonia for transforming ellipsoidal heights to the BK77 system in Estonia (The Minister of the Environment's… 2004, 2008).

The focus here is determination of model Est-Geoid2003 including later updates and on re-evaluation due to the availability of new precise levelling data.

2. Processing of Gravimetric Data

A great emphasis was laid on the gathering and inclusion in the geoid calculations of gravimetric data. Data was searched at different archives and then digitized and transformed to the present system of absolute gravity, and its accuracy was assessed. Gravimetric data from various regions that had not been used previously for Estonia's geoid calculation was included (Fig. 1): from some regions of Russia; from the big lakes of Peipsi, Pihkva and Võrtsjärv (~700 points, survey on the ice 1987); from the Gulf of Riga (~1,600 points, bottom survey 1967–68); and from the detailed measurement project performed by the Geological Survey of Estonia (~120,000 points).

The quality of the Gulf of Riga and Lake Peipsi gravity data was checked by ice measurements during 2008–2010 (Fig. 2). More as 30 points were measured. Originally declared accuracy (better than 0.5 mGal) was identified (Oja *et al.* 2009).

The gravimetric network of Estonia consists of about 300 points measured over the last ten years by State Land Board. In the research, use was made of gravimetric data for approximately 136.000 points (apart from additional points from the KMS-NKG database), which provided the basis for gravimetric geoid calculations (Figs. 1, 3). Dense data set from Geological Survey around Tartu was added in 2010 (Fig. 4).

Due to the fact that the quality of the gravity surveys from the 1950s is low, a new re-measuring campaign is under way beginning from 2006 in cooperation with different institutions (Tallinn Technical University, Estonian Land Board, Estonian University of Life Sciences (ELS), etc.). The gravimeters Scintrex and La-coste Romberg have been used. A significant portion of south Estonia has already been covered and a profusion of data is being processed (Fig. 4). This makes it possible to improve the gravimetric solution even further in the near future.

3. Calculation of the gravimetric geoid

The remove-restore method using FFT was used for precise gravimetric geoid determination. It was the best method considering its speed in processing large amounts of data. The original gravity data (free air anomalies) was gridded by means of collocation using data points inside a 20-km radius. Then, the global field effect was removed before Stokes' integration. The interpolation of pure free air anomalies into the grid was not perfect due to the terrain effect. However, the terrain was low and rather smooth in the area (normally lower than 150 m), which reduced interpolation errors. The application of Bouquer anomalies for gridding sets requires a precise digital elevation model. This is being built by the Land Board using laser scanning but not yet complete.

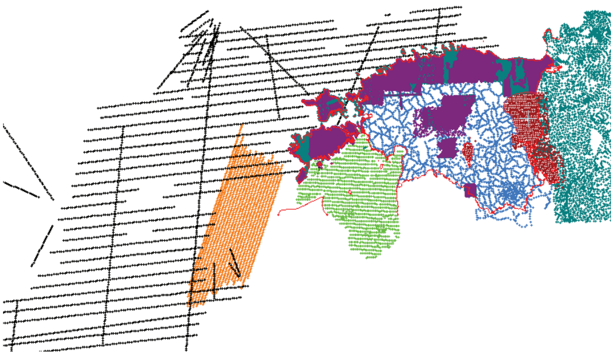

Fig. 1. Gravimetric data collected for the research before 2005

Fig. 2. Gravity survey made on ice by Kristina Türk in 2010

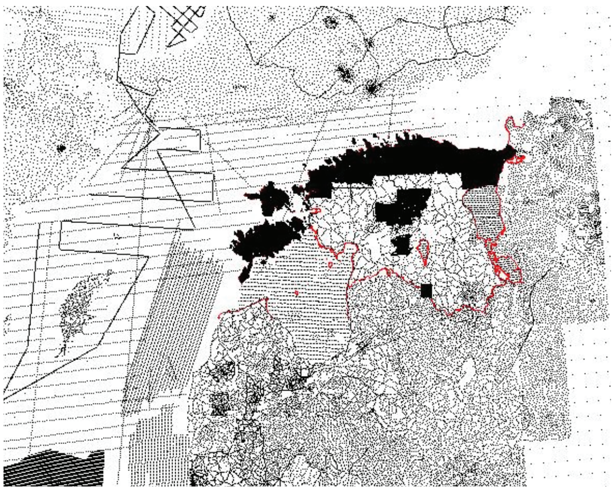

Fig. 3. Gravimetric data coverage in 2003–2008

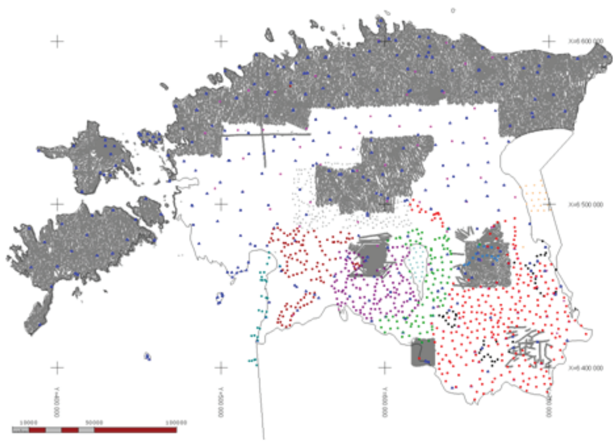

Fig. 4. A new gravity survey in south Estonia, 2007–2010

The gravimetric geoid (Fig. 5) was calculated as an approximately 3-km net. The integration area was 18°–30° longitude and 56°–62° latitude. Originally, the EGM96 global model was used for converting long wavelength signals into local gravity.

Subsequently, several new combined GRACE satellite models were tested. For example, the global geopotential model GRACE05C (2008) fits with GPS-levelling geoid much better (Fig. 6). Standard deviation is 11 cm while 23 cm in use of EGM96. Change to GRACE05C results in an absolute shift of approximately 10–15 cm for the final gravimetric geoid (Fig. 7). However, there was no big influence on the gravimetric geoid surface tilt, probably due to the good coverage of the terrestrial gravimetric data.

In fact, the calculations resulted height anomalies due to uncorrected free air anomalies used as input data. This means that the effect of downward continuation was ignored, which significantly simplifies the procedure.

Quasigeoid values compared to geoid undulation are normally less than 1 cm in the region and can be ignored here because the ultimate goal is a fitted geoid model. Furthermore, the terrain effect is very small and smooth due to small heights in the area and was not taken into account.

After including in the calculations the gravimetric data collected, the gravimetric geoid no longer had a big tilt relative to the geometric geoid (Fig. 8), as was previously the case (Jürgenson 2001).

The standard deviation between the gravimetric geoid and GPS-levelling points was 2.3 cm across Estonia, from –48 cm to –55 cm. The normal height of Kihnu Island was updated by ELS using water level monitoring in 2006–2008 (Liibusk, Jürgenson 2008) into RGT point 501. Almost 12 cm wrong height value from catalogue (1977) caused distortions in 2003 (Jürgenson 2004).

Thus, the processing of the gravimetric and GPS-levelling data yielded a similar value for the topography of Estonian geoid.

In Fig. 9 we can see the change in the gravimetric geoid while the global field is replaced from EGM96 to GRACE05C. The absolute level was changed according to Fig. 7, but the standard deviation increased from 2.3 cm to 2.7 cm. This may have been caused by different terrestrial gravity used inside global models. Also gravimetric data from Geological Survey were added from Tartu region (Fig. 4).

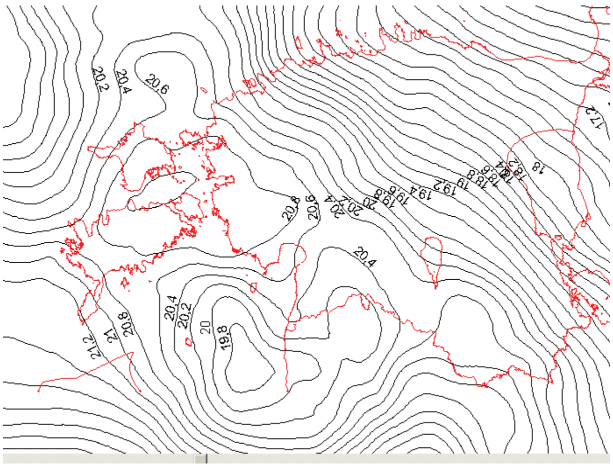

Fig. 5. Gravimetric geoid. Contour interval 20 cm, units in m

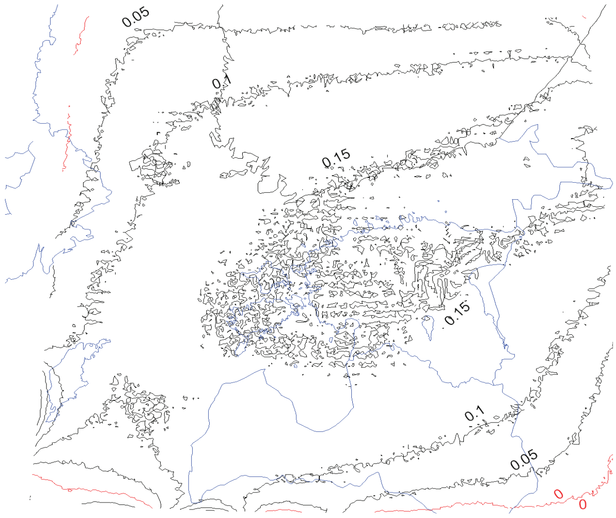

Fig. 7. Differences (m) in the gravimetric geoid calculated using the combined model EGM96 or GRACE05c

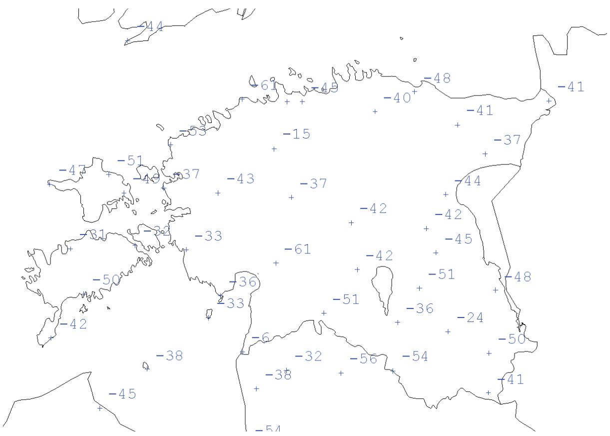

Fig. 6. Differences between GRACE satellite model EIGEN05C and GPS-levelling points

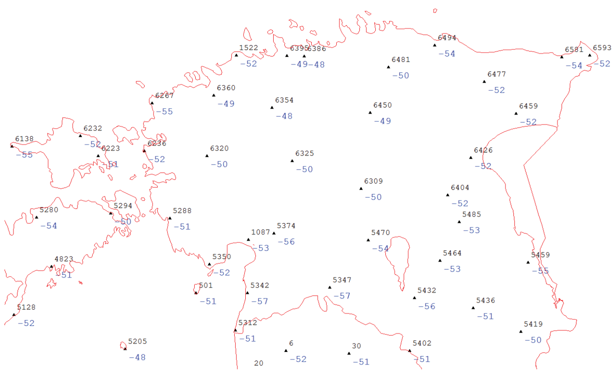

Fig. 8. Differences of the gravimetric (1) and geometric (2) geoid (2–1) in cm

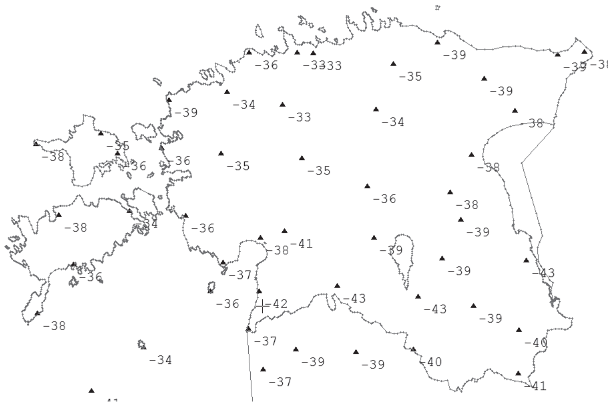

Fig. 9. Differences in the gravimetric (1) and geometric (2) geoid (2–1) in cm. The GRACE05c model was included in geoid computations

4. Calculation of the fitted geoid model

The surface of the calculated gravimetric geoid was fitted by GPS-levelling points. The procedure was as follows: up to 50 RGP geometric geoid point heights N_g were determined based on data from geodetic networks.

$$N_g = H - h,$$

where H is a BK77 normal height from the 1977 catalogs and h is a Euref-Est97 geodetic height. In addition, the height of the physical geoid N_f was interpolated to the same points. After that, we could calculate the difference between the two geoid heights as follows:

$$dN = N_g - N_f.$$

Then, the dN values were gridded from 50 points. The result was a 3-km dN grid. During the gridding process, the dN values of 11 closest points were used, as were the distances for weights. The gridding function is not linear; however, the polynomial degree is not very high either. Distances of up to 20 km from fitting points usually result in residuals of up to 1 cm on the fitting points.

After that, the dN grid was just added to the gravimetric geoid grid. This resulted in the determination of the so-called fitted geoid, or the reference model, which with maximum precision reflects the difference on Estonian territory between the normal heights of BK77 and the Euref-Est97 ellipsoidal heights of the basic geodetic network RGP. The model was originally named Est-Geoid2003 (Fig. 10), and the name has remained in spite of minor improvements. Even the gravimetric geoid changes a little, the fitted model remains almost the same (inside 1 cm), if fitting points remains the same.

The accuracy of RGP heights has been declared to be higher than 1 cm (Rüdja 2002). The reason for the high accuracy is that the RGP points were established and measured based on the same principles and in the same campaign (in 1997). In the Est-Geoid2003 fitting process, use was only made of RGP points, the normal heights of which were measured from I and II order benchmarks. Subsequently, the Russian II order levelling was promoted to I order (Torim 1994). 26 points were levelled in 1998 by State Land Board. In order to densify the existing fitting points network, the measurement of 13 additional RGP points was organized (using II order benchmarks) across Estonia in 2001–2003 by ELS (Jürgenson 2003). Perjatsi (6581) was levelled in 2004 (Ostrovskaja 2004).

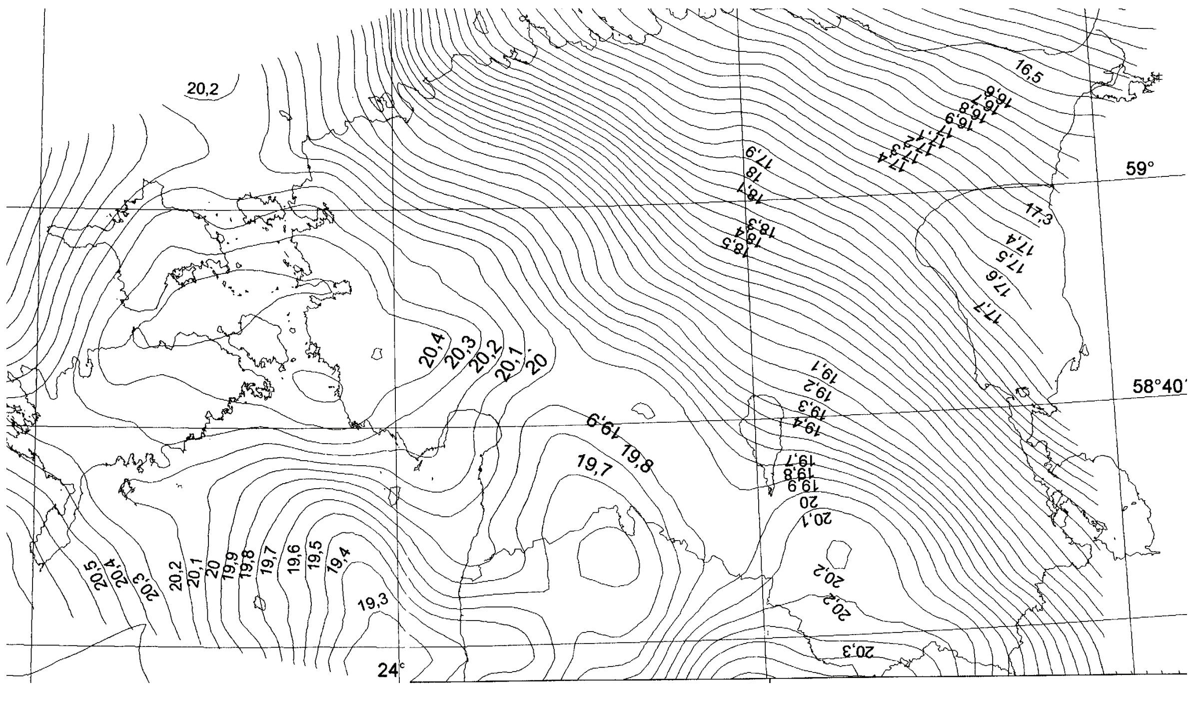

Fig. 10. Fitted geoid model Est-Geoid2003, contour interval is 0.1 m

5. Upgrade Est-Geoid2003 in 2010

Five new fitting points were added in 2010, their heights being calculated from new precise levelling lines (Planserk 2004–2009) from the closest BK77 benchmarks. In the course of the computations, a problem arose with the stability of the benchmarks. In some cases, three closest benchmarks yielded different results by up to 1–2 cm in BK77. From preliminary tests we knew that the cause of the problem was not new levelling. Polygon misclosures are less than 3 mm per hundreds km. This meant that the stability of older wall benchmarks was problematic.

This led us to the realisation that reaching a 0.5-cm fitted geoid by using wall benchmarks was not a realistic task. A better solution appears to be the computation of BK77 heights from the closest deep benchmarks (usually 20–40 km apart).

The total number of the fitting points is currently 50 (including 4 in Finland and 5 in Latvia), which is almost sufficient for an area the size of Estonia. After the final adjustment of the new levelling, the number of levelled RGP points will increase to approximately 100; probably, not all of them need to be included to fitting points. Obviously, the new datum will also change the fitted geoid model.

6. The accuracy of the model Est-Geoid2003

The accuracy of the Est-Geoid2003 was tested using points taken from different measurement campaigns.

Specific GPS-levelling works were designed for checking the model in 2003. An ellipsoidal height was measured to a temporary point, which was connected to a II order levelling benchmark. Additionally, points with II order normal heights from the GPS densification network RGT (ellipsoidal heights of a III order GPS net also have a 1–2 cm accuracy, Rüdja 2002) were used for evaluation. About 75 checking points yielded an accuracy of 1.3 cm as rms.

An evaluation using RGT points with III and IV order normal heights revealed a mean-square error of 1.9 cm. However, the error was attributable to a lower accuracy of the control points themselves.

A special study was performed using 37 RGT points that were levelled due to works on the geodetic network of local towns using III order levelling (in 1998–2004). Normally, there were 3 points per town. These points revealed a mean-square error of 1.3 cm (Ostrovskaja 2004).

Checking of the Est-Geoid2003 model using connections from new precise levelling

Obviously, an authentic test of model accuracy would require points belonging to the same accuracy level as those used for the fitting of the model. The new precise levelling net (Fig. 12) is part of the integrated network (Fig. 11), with some new polygons added compared to the original plan. New high-precision checking points (RGP) became available using connections from a new precise levelling campaign (levelling 2001–2010). Levelling is financed by Land Board. In the test under study, the normal heights of the 17 RGP points (Fig. 13) were calculated from the closest II or I order benchmarks. The ellipsoidal heights came from the RGP I and II order campaigns (1998) again. Datum unification was not performed to maintain conformity with official realisations. The purpose of the Est-Geoid2003 model is to be a link between the present realisations.

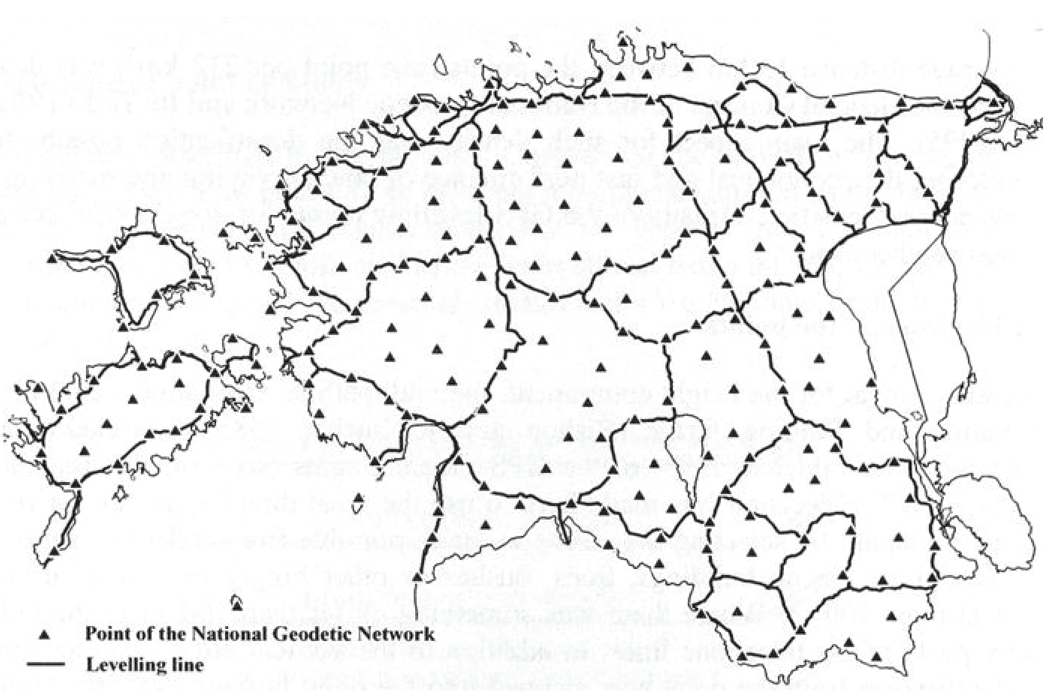

Fig. 11. Integrated geodetic network. I and II order GPS networks and precise levelling lines (partially from old levelling, Rüdja 2004)

Geoid heights from 17 highest-accuracy GPS-levelling points were compared with those of Est-Geoid2003 (Fig. 13). These test points were not used in the modelling process. We can see from figure 13 that geoid differences between the geometric geoid and the Est-Geoid2003 model are less than 2.3. Mean square erros is 1.2 cm. Unfortunately results from islands will come a little later.

It appears from the different tests that the accuracy of the Est-Geoid2003 model is 1–2 cm relative to I and II order levelling networks of BK77 and to the RGP network. In fact, Est-Geoid2003 is in many cases more accurate than the height values of IV order benchmarks and suitable for everyday geodesy. More and more heights determinations are made using satellite positioning and geoid model. 0.5 cm geoid model is a goal to achieve during next years. This will be possible after updating gravity database and levelling network.

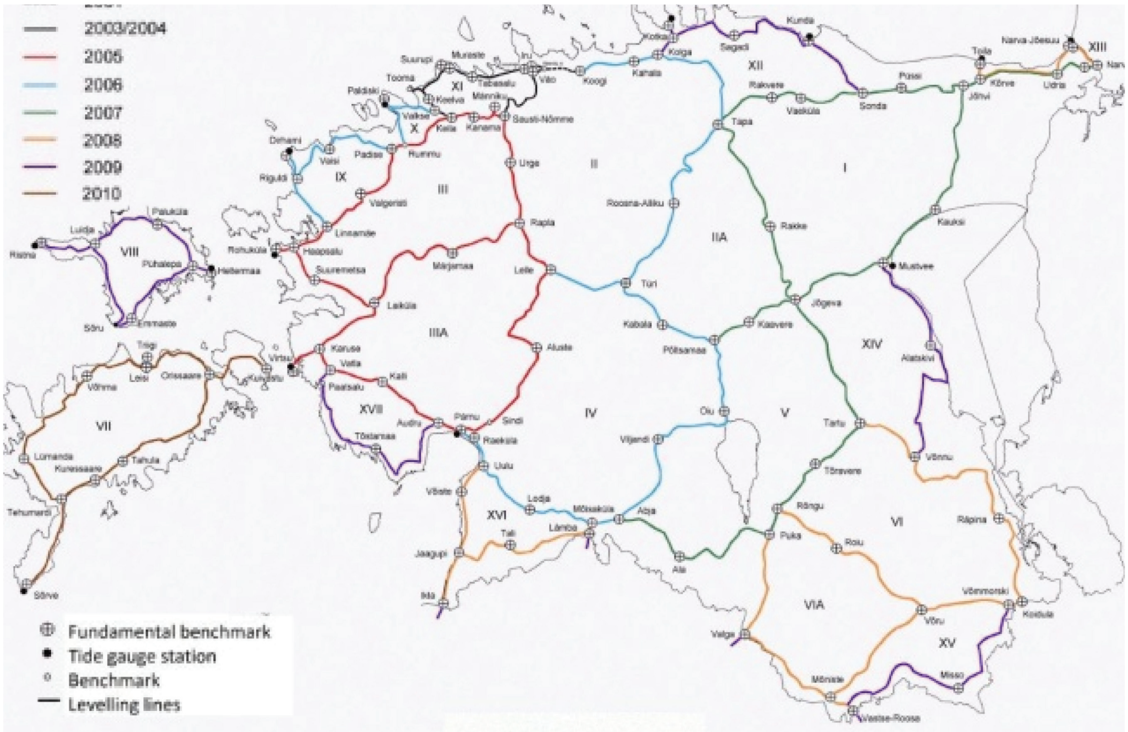

Fig. 12. Polygons of the new levelling 2001–2010

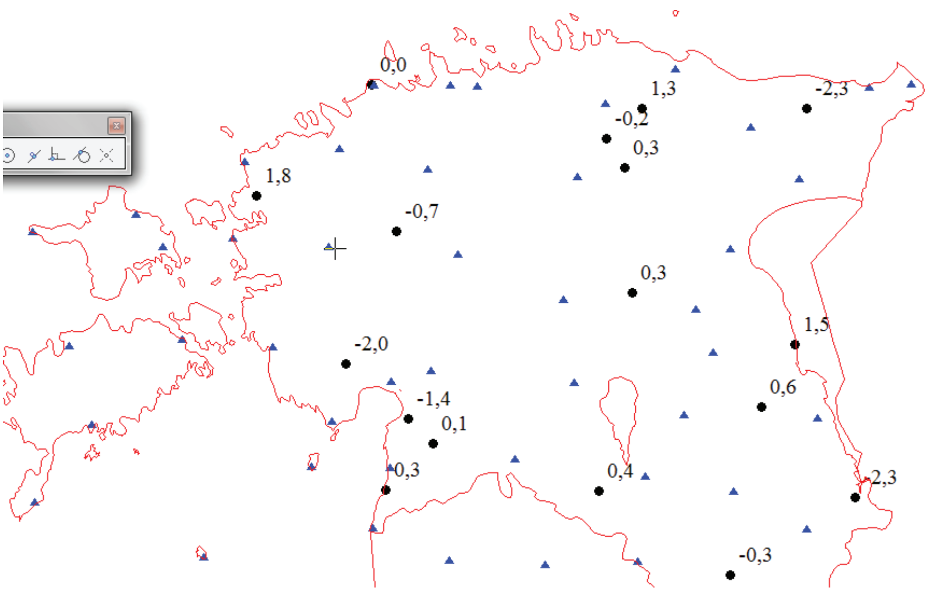

Fig. 13. New high-precision checking points (circles) among 50 fitting points of the Est-geoid2003 model

7. Conclusions

From the gravimetric data gathered, a gravimetric geoid was calculated for Estonia as an approximately 3-km net using the FFT method (Fig. 5). By now, comparisons of the Estonian gravimetric geoid with new global models have been made. It is clearly observable that newer global models are much more accurate in our district than earlier versions. For example, the combined model GRACE05c yields a standard deviation of 11 cm compared to geoid heights from GPS levelling points.

After including in the calculations all the new gravimetric data collected, the gravimetric geoid no longer had a remarkable tilt relative to the geometric geoid (Fig. 6). The standard deviation 2.7 cm using GRACE05C as reference model.

The surface of the calculated gravimetric geoid was fitted by GPS-levelling points. This resulted in the determination of the so-called fitted geoid, or the reference model, which reflects the difference on Estonian territory between the normal heights of BK77 and the ellipsoidal heights of the basic geodetic network RGP. The datum difference was not taken into account in the fitting process.

The accuracy of the fitted model Est-Geoid2003 is 1.3 cm based on test points with normal heights from II order benchmarks. The result was obtained using geodetic densification networks (RGT) points with normal heights levelled from the nearest II order benchmarks. Similar accuracy appeared from 37 test point of local towns as well (1.3 cm).

Particularly interesting are the results from new precise levelling (Figs. 12, 13), according to which the model errors are less than 1.2 cm as rms. Here 17 test points are analyzed from the same geodetic base networks as fitting points (RGP) but independent of them.

The instability until of wall benchmarks was noticed while determining the normal heights to RGP points from several benchmarks. This limits the ultimate accuracy of fitted geoid to a level not higher than 1–2 cm. More precise normal heights will be obtained from new levelling after uniformal adjustment.

Acknowledgements

This paper has been supported by ETF grants No. 5731 and 7356, the Estonian Land Board and the NKG gravity database.

References

Jürgenson, H. 2001. Geoid Computations in Estonia, *IGeS Bulletin* 11. Milano.

Jürgenson, H. 2003. *Determination of Estonian Precision Geoid*: Thesis for Doctor of Philosophy in Engineering. Tartu, Manuscript.

Jürgenson, H.; Kall, T. 2004. The Difference between the N60 and BK77 Height Systems, *Nordic Journal of Surveying and Real Estate Research* 72–85.

Ellmann, A. 2004. *The geoid for the Baltic countries determined by the least squares modification of Stokes' formula*: Thesis for Doctor of Philosophy in Engineering. KTH – the Royal Institute of Technology.

Liibusk, A.; Jürgenson, H. 2008. Connection of Gulf of Riga Islands to the Baltic Heights System 1977, in *7th International Conference on Environmental Engineering*: Selected papers. Vilnius, Lithuania, May 22–23, 2008. Vilnius: Technika, 1378–1385.

Oja, T.; Bloom, A.; Gruno, A. 2009. *Raskuskiirenduse mõõdistamine Võrts- ja Peipsi järvel. a. Gravity surveying on lakes Peipsi and Võrtsjärv*: Report. 2009. Tallinn, Land Board archive.

Ostrovskaja, K. 2004. *Mudeli Est-Geoid2003 kontrollimine* (Check of the model Est-Geoid2003): Thesis. Tartu, ELS. 87 p.

Planserk AS (2004–2009): Geodeetiliste tööde aruanded: Reports of geodetic field Works. Tallinn.

Rüdja, A. 2002. Realisation of the ETRS89 in Estonia, in *Proceedings of the 14th General Meeting of the Nordic Geodetic Commission*, Espoo, Finland, October 1–5, 2002. Kirkkonummi, 211–221.

The Minister of the Environment's regulation "The Establishment of the Geodetic System", 2004, 2008. Tallinn, RTL, 18.02.2004, 17, 267.

Torim, A. 1994. *I, II ja III klassi nivelleerimise eeskiri (Rules of I, II and III order levelling)*. Eesti Vabariigi Maa-amet. Tõravere.

Tscherning, C. C.; Forsberg, R.; Knudsen, P. 1992. The GRAVSOFT package for geoid determination, in *Proc. 1st Continental Workshop on the Geoid in Europe*. Praha.

Harli JÜRGENSON. Assoc. Prof., Estonian University of Life Sciences, Dept of Geomatics, Ph +3727313, Fax +372313156.

He holds (since 2003) Phd degree from the Estonian University of Life Sciences in Tartu. PhD topic was "Estonian precision geoid". Over 18 presentations in international conferences and seminars, author of over 25 publications inferred magazine articles, conference proceedings, technical reports. As a result of his research plane coordinate system Lambert-Est and fitted geoid model Est-Geoid2003 have been taken into use in Estonia.

Research interests: geoid modelling, height datums, gravity field, plane coordinate systems.

Kristina TÜRK. Doctoral student, Estonian University of Life Sciences, Dept of Geomatics, Ph +3727313, Fax +372313156.

Research interests: Estonian gravity field modelling, LaCoste & Romberg gravimeters.

Jüri RANDJÄRV. Prof. emeritus, Estonian University of Life Sciences, Dept of Geomatics, Ph +3727313, Fax +372313156.

Over 25 presentations in international conferences and seminars, author of over 30 publications.

Research interests: precise levelling, land uplift.