JOURNAL OF CIVIL ENGINEERING AND MANAGEMENT
ISSN 1392-3730 print/ISSN 1822-3605 online
2013 Volume 19(1): 9–15
doi:10.3846/13923730.2012.734857

RESEARCH ON MODEL FITTING AND STRENGTH CHARACTERISTICS OF CRITICAL STATE FOR EXPANSIVE SOIL

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Received 28 Apr. 2011; accepted 08 Jun. 2011

Abstract. According to Mengzi expansive soil, consolidated drained tests and undrained tests are carried on under saturated and remoulded conditions. The stress-strain characteristics of saturated soil are researched systematically under different confining pressure, initial dry density, initial water content, shearing rate and drainage condition. The inherent unity of diversity of shearing strength for the same samples measured by different experimental methods is indicated according to the normalization of critical state test results. And the failure lines in $p'-q-\nu$ space of remoulded saturated expansive soil under consolidated drained and undrained conditions are attained. The hyperbolic curve model can fit well the weak hardening stress-strain curves and the exponential curve model can fit the weak softening stress-strain curves. The test results can provide technical parameters and theoretical help for shearing strength variation of slope during rainfall and strength state of soil structure in normal water level.

Keywords: expansive soil, critical state, remoulded soil, normalization.

1. Introduction

Undrained strength indices are often used in analysis of strength and stability for the difficult in the determination of excess pore water pressure in practical engineering. Schofield (2006) suggested that the critical strength should be adopted for design basis. The K_0 compression curves of normally consolidated soil and critical state lines can be simplified to be parallel lines in $e-\lg p'$ coordinate system. Wood (1990) proposed that the stress ratio of critical state met certain relationships in triaxial compression and tension condition. Roscoe and Burland (1968) pointed out that there was a unique relationship, which did not change with stress path, among p, q, e of normally consolidated clay and weak consolidated remoulded clay. Parry (1960) indicated that drained triaxial shearing test results going through different stress path under the same confining pressure would achieve the same critical state for normally consolidated soil and overconsolidated soil. Chen and Saleeb (2005) proposed that the shearing strength of samples with the same initial void ratio would tend to be uniform in undrained shearing tests. Liu *et al.* (2006) indicated that critical stated model was capable of simulating strain hardening, softening, normal dilatancy and stress-path dependency of interface between sandy soil and structures during shearing. The unified modeling of the behavior of interfaces with different roughness, different density of soil and different normal pressures using the concept of critical state soil mechanics is also successfully attempted. Gitau *et al.* (2006) applied critical state soil mechanics to study the stress-strain behaviour of the luvisols using triaxial tests under laboratory conditions. An exponential model used to fit the deviatoric stress-axial strain test data accurately predicted the soil shear strength at the critical states. Saffih-Hdadi *et al.* (2009) proposed that confined compression tests were performed on remoulded soils at different initial bulk densities and water contents. Good linear regression was obtained between soil precompression stress, the compression index, initial water content, initial bulk density and soil texture. Suebsuk *et al.* (2010) introduced a plastic potential to account for the influence of structure on the plastic strain direction for both hardening and softening behaviours. Muhunthan and Worthen (2011) pointed out that the shear strength and deformation behavior of soil was quite sensitive to the combination of changes in volume and confining stress.

Remoulded expansive soils in Mengzi area are used in consolidated drained tests (CD) and consolidated undrained tests to discuss the saturated strength characteristics. The stress-strain characteristics of saturated samples under different consolidated confining pressure σ_3, initial dry density ρ_d, initial water content ω_0, shearing rate υ and drainage condition are researched and fitted. The test results can provide technical parameters and theoretical help for shearing strength variation of slope during rainfall and strength state of soil structure in normal water level.

2. Test method

Test apparatus is an automatic triaxial shearing apparatus which is produced by KTG company. The apparatus equips with programmable stepless variable speed control and stepping motor loading. Samples are selected at Mengzi County in Yunnan province. Qu *et al.* (2000) indicated that the expansive soil in the area belongs to eluvial class fissured clay. Mineral component of clay include montmorillonite, illite and kaolinite. The content of montmorillonite, which is higher, is about 22% to 48%. The free swelling ratio of samples is about 30% to 60%. The highest is up to 130%. The physical and mechanical indices of samples are presented in Table 1.

The specific test method is as following: 1 – the expansive soil is weathered and grinded. The soil particles are filtered by geotechnical sieve with aperture of 2 mm diameter. Some water is added to reach certain water content and enclosed more than 48 hours; 2 – remoulded samples are prepared as diameter of 39.1 mm and 80 mm height. The demands of consolidated drained test are 20% of initial water content, 1.5 g/cm^3 of dry density, 0.012%/min and 0.007%/min of shearing rate. The demands of consolidated undrained test are 16% and 18% of initial water content, 1.5 g/cm^3 and 1.7 g/cm^3 of dry density, 0.1%/min and 0.05%/min of shearing rate. In Table 2, SS means shearing strain softening and SH means shearing strain hardening; 3 – samples are laid in vacuum chamber and pumped down to be saturated more than 6 hours. The saturated sample is put in KTG automatic triaxial shearing apparatus. When value B is equal or greater than 0.95, it means the sample is saturated by back pressure. Then the drained of undrained test is carried on after the isotropic consolidation.

3. Analysis of test results

3.1. Effect of confining pressure

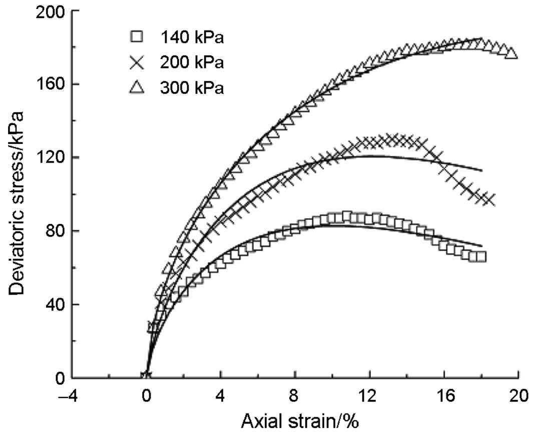
The relationship between stress and strain is presented in Fig. 1. The bigger the confining pressure is, the bigger the shearing strength is. The sample which has bigger shearing strength is easy to exhibit a kind of strain softening characteristic. The phase transition behavior from strain hardening to strain softening appears gradually with the confining pressure increasing. The phenomenon of shear dilatation occurs easier when the density of consolidated sample is larger and larger with consolidated pressure increasing. The topsoil is easy to collapse for the lower confining pressure and shear strength in slope of expansive soil.

3.2. Effect of initial dry density

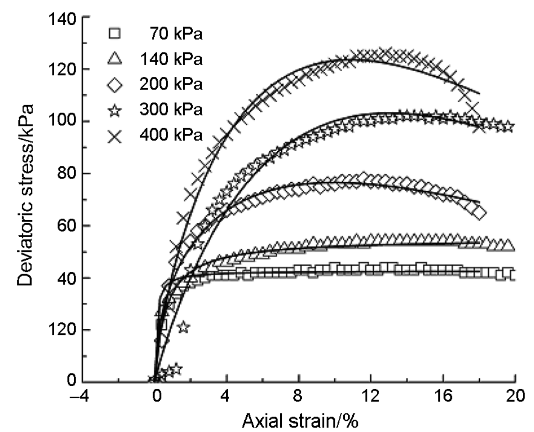
It has great effect for initial dry density to stress-strain characteristics of samples under the same initial water content and shearing rate in CU tests. The steady state line of sample with the bigger initial dry density is underneath the line of sample with the smaller initial dry density in the relationship between average effective normal stress and specific volume in Fig. 2. The two steady state lines of samples with different dry density show the parallel relationship. The bigger the initial dry density is, the bigger the shearing strength is. The stress-strain behaviors of samples change from strain hardening to strain softening with dry density increasing. It has great guiding significance for the change of failure mode in engineering practice of expansive soil.

3.3. Effect of initial water content

It has great effect for initial water content to stress-strain characteristics of samples under the same initial dry density and shearing rate in CU tests. The steady state line of



a) CD1



b) CU2

Fig. 1. Fitting stress-strain curves for partial samples

Table 1. Physical and mechanical indices of expansive soil samples

Type	Natural w_0, %	Liquid limit, %	Plastic limit, %	Plastic index	< 0.002 mm content, %	Maximum dry density, $g \cdot cm^{-3}$	Optimum ω_0, %	Free swelling ratio, %	Swelling force, kPa
Redish brown	20.5	57.1	22.1	35.0	22.1	1.70	17.0	75	330.3

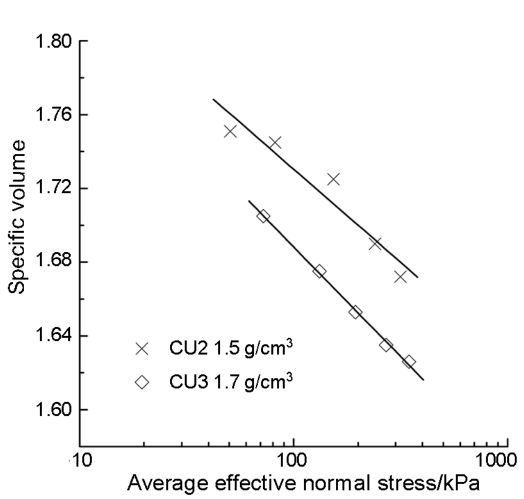
Table 2. Fitting parameters for stress-strain curves

Tpye	w_0, %	ρ_d, g/cm^{-3}	υ, % min^{-1}	σ_3, kPa	Failure mode	Parameters of hardening model					Parameters of softening model		
						a	b	E_i	k	n	M	N	L
CD1	20	1.5	0.012	140	SS						33.5472	0.6897	–0.0684
				200	SS						40.6816	0.7264	–0.0600
				300	SS						51.5609	0.5885	–0.0236
CD2	20	1.5	0.007	70	SH	0.0620	0.0089	16.1290	0.2208	0.9297			
				140	SH	0.0343	0.0051	29.1550					
				200	SH	0.0236	0.0039	42.3730					
				300	SH	0.0161	0.0031	62.1120					
CU1	16	1.5	0.100	70	SH	0.0097	0.0291	102.7749	0.7022	0.0209			
				140	SH	0.0142	0.0136	70.4230					
				200	SS						46.3532	0.3473	–0.0298
				300	SS						40.2578	0.5524	–0.0452
				400	SS						39.9054	0.6807	–0.0561
CU2	16	1.5	0.050	70	SH	0.0023	0.0234	434.7830	0.7022	0.0209			
				140	SH	0.0111	0.0181	90.0900					
				200	SS						40.9723	0.4811	–0.0483
				300	SS						24.5009	0.9164	–0.0702
				400	SS						44.4514	0.7365	–0.0676
CU3	16	1.7	0.050	70	SS						52.4429	0.2817	–0.0351
				140	SS						48.9973	0.5491	–0.0681
				200	SS						58.5159	0.5645	–0.0773
				300	SS						34.8325	0.8699	–0.0702
				400	SS						79.8355	0.3974	–0.0307
CU4	18	1.5	0.050	100	SH	0.0279	0.0264	35.8420	0.7022	0.0209			
				200	SH	0.0127	0.0159	78.7400					
				300	SS						51.2766	0.2956	–0.0168
				400	SS						13.2140	1.4627	–0.1338

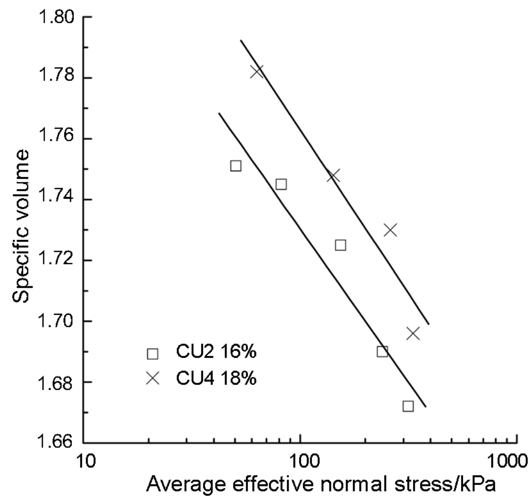
sample with the bigger initial water content is above the line of sample with the smaller initial water content in Fig. 2. The two steady state lines of samples with different water content show the parallel relationship. The lower the initial water content is, the bigger the shearing strength is.

3.4. Effect of shearing rate

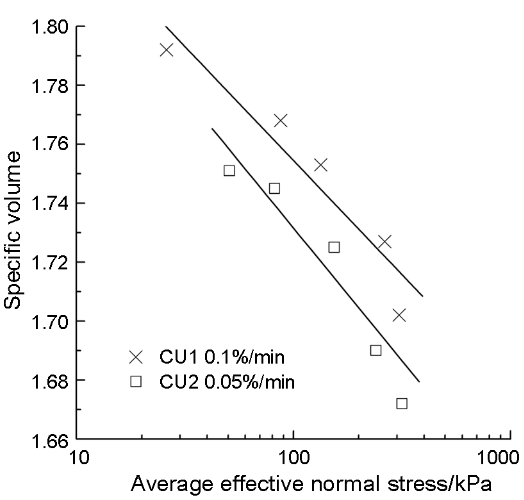
It has great effect for shearing rate to stress-strain characteristics of samples under the same initial dry density and water content in CU and CD tests. The results are presented in Fig. 2c and 2d. The steady state line of sample with the bigger shearing rate is above the line of sample with the smaller shearing rate. The two steady state lines of samples with different shearing rate show the parallel relationship. The larger the shearing rate is, the larger the shear strength of sample is in CU test. The larger the shearing rate is, the smaller the shear strength of sample is in the CD test. The rule is just opposite for shearing drainage condition. The higher shearing rate causes strain softening phenomenon and the lower shearing rate causes strain hardening of sample. The phenomenon indicates that the bifurcation appears in shearing process. One is that the deformation of sample expands sequentially and tends to be hardening condition. The other is that shearing zone appears and tends to softening in macroscopic view. The long-axis direction of soil particle is gradually perpendicular to direction of maximum principal stress to increase the contract area inter particles in shearing process when the shearing rate is smaller. At the result, the hardening phenomenon behaves gradually. If all the particles achieve the oriented arrangement and hardening reaches a limit, the compressive deformation of sample will emerge uniformly. When shearing rate is bigger, the main deformation will be limited in shearing zone and the particles directionally arrange along shearing zone. The main reason is that there's not enough time for soil particles to make hardening arrangement. The oriented arrangement of particles often begins from the weak points inside the sample and then extends gradually through a shearing zone.



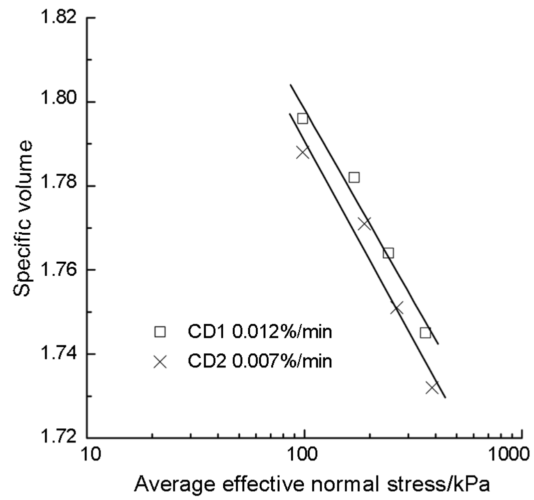
a) Different initial dry density



b) Different initial water content



c) Different shearing rate for CU tests



d) Different shearing rate for CD tests

Fig. 2. Relationship between average effective normal stress and specific volume under different parameters

3.5. Effect of drainage condition

Alarcon-Guzman *et al.* (1988) indicated that it had different effect for different drainage conditions to steady state. Yamamuro and Lade (1998) found that the steady state line of silt under drainage condition and undrainage condition was the same line within the limit from 200 kPa to 1000 kPa confining pressure. The steady state line under drainage condition was still higher the line under undrainge condition below 200 kPa pressure. Yang (2000) concluded that it had a certain effect to steady state for drainage condition of weathered granite and weathered volcanic in Hong Kong. The steady state line of sample under drainage condition which exhibits a higher steady state is above the line under undrainage condition in Fig. 3. The effect of drainage condition to triaxial shearing test exhibits mainly the influence of excess pore water pressure during shearing test. The stain hardening behavior is easier to appear under drainage condition than undrainage condition.

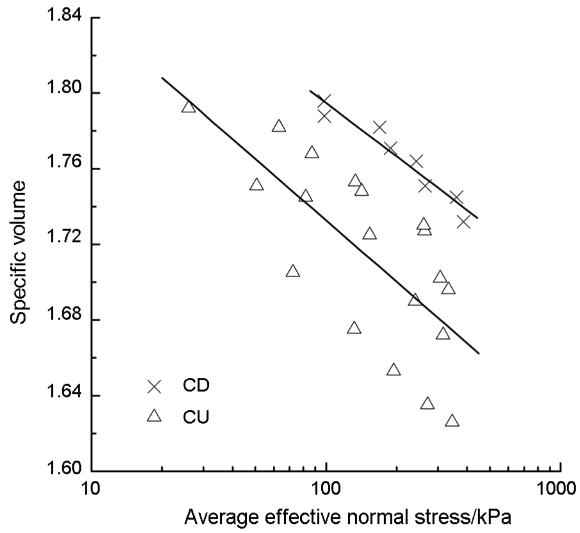


Fig. 3. Comparison between p' and v under different drainage conditions

3.6. Treatment of normalization

Li *et al.* (2008) measured the mechanical strength of expansive soil by different test methods to attain the uniform rule. Different sample dimension and test methods will cause the differences in mechanical strength measuring of soil. The samples used in triaxial tests have three kinds of diameters: 39.1 mm, 50.0 mm and 61.8 mm. The different sample dimension will cause different test results. It has great differences to compare shearing test results of indoor samples and big samples in field. The mechanical parameters tested by different apparatuses all have differences attributing to the differences of characteristics of shear plane and stress conditions. So the relationship between average effective normal stress and specific volume is not the only one. It depends on some factors such as initial water content, initial dry density, drainage condition, shearing rate, and so on. The normalization method can be used to achieve the unique relationship between average effective normal stress and specific volume. The initial dry density and water content are used to be normalization parameters in the same shearing rate condition as showed in Eq. (1). The linear relationship after normalization can be expressed by the formula $v = \Gamma - \lambda \ln p'$ as showed in Fig. 4:

$$e = e_c \rho_n \omega_n, \tag{1}$$

where: $\rho_n = \rho_0 / \rho_{op}$, $\omega_n = 1 + \dfrac{\omega_{op} - \omega_0}{5\omega_{op}}$, e is void ratio; e_c is void ratio after consolidation; ρ_0 is initial dry density; ρ_{op} is the optimum dry density; ρ_{op} = 1.70 g/cm^3; ω_0 is initial water content; ω_{op} is the optimum water content; ω_{op} = 17%. The linear relationship is fitted by the formula: $v = 1.7726 - 0.0389 \ln p'$. Here: v – specific volume; p' – average effective normal stress. The fitting parameters Γ = 1.7726; λ = 0.0389.

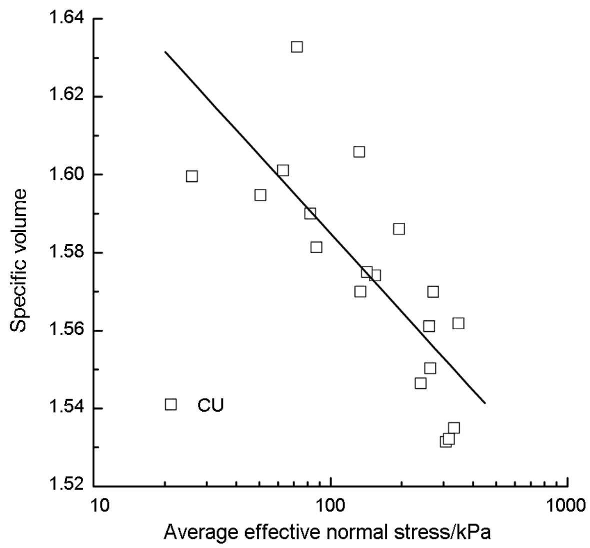


Fig. 4. Relationship between p' and v after normalization

4. Fitting of stress-strain model

In CU tests, the stress-stain relationship of sample pertains to weak hardening when the consolidated pressure is smaller than 200 kPa. And the relationship pertains to weak softening when the pressure is bigger than 200 kPa. In CD tests, the stress-stain relationship of sample belongs to weak hardening when the shearing rate is smaller. And the relationship belongs to weak softening when the shearing rate is larger.

The hyperbolic model (2) can be used to simulate the weak hardening stress-strain relationship as showed in Fig. 1 and the fitting parameters are showed in Table 2:

$$q = \sigma_1 - \sigma_3 = \frac{\varepsilon_a}{a + b\varepsilon_a};$$

$$\frac{\varepsilon_a}{q} = a + b\varepsilon_a, \tag{2}$$

where: q is deviatoric stress; σ_1 is major principal stress; σ_3 is minor principal stress; ε_a is axial strain; a, b are fitting parameters. They can be attained by stress-strain curves of conventional triaxial tests.

The tangent modulus E_t is attained by derivation of q with respect to ε_a as showed in Eq. (3):

$$E_t = \frac{dq}{d\varepsilon_a} = \frac{a}{(a+b\varepsilon_a)^2}. \tag{3}$$

The young modulus:

$$E_i = \left(\frac{dq}{d\varepsilon_a}\right)_{\varepsilon_a \to 0} = \frac{1}{a}. \tag{4}$$

The peak deviatoric stress:

$$q_p = q_{\varepsilon_a \to \infty} = \frac{1}{b}. \tag{5}$$

The initial young modulus E_i depends on initial confining pressure σ_3 and initial dry density ρ_d. Janbu (1963) suggested that initial young modulus could be expressed by the Eq. (6) considering void ratio:

$$E_i = kP_a\left(\frac{p_0}{p_a}\right)^n, \tag{6}$$

where k and n are test parameters. They can be attained by the power function relationship between $\frac{E_i}{p_a}$ and $\frac{p_0}{p_a}$. P_a is standard atmosphere.

Exponential model (7) can be used to simulate weak softening stress-strain curves as showed in Fig. 1. The fitting parameters are showed in Table 2:

$$q = \sigma_1 - \sigma_3 = M\varepsilon_a^N e^{L\varepsilon_a}. \tag{7}$$

The Eq. (8) is attained by taking logarithm of both sides of Eq. (7):

$$\lg q = A + N\lg\varepsilon_a + B\varepsilon_a, \tag{8}$$

where: $A = \lg M$, $B = L\lg e$, M, N, L, A, B are fitting parameters.

The tangent modulus E_t is attained by derivation of Eq. (8) with respect to ε_a as showed in Eq. (9):

$$E_t = \frac{dq}{d\varepsilon_a} = q\left(\frac{N}{\varepsilon_a} + L\right). \tag{9}$$

Young modulus:

$$E_i = \left(\frac{dq}{d\varepsilon_a}\right)_{\varepsilon_a \to 0} = \infty. \tag{10}$$

5. Conclusions

According to Mengzi expansive soil, the saturated strength tests of consolidated drainage (CD) and undrainage (CU) are carried on. The mainly conclusions are as following:

1. The mechanical characteristics of Mengzi saturated soil are researched in CD and CU conditions by using theory of critical state. It has great influence for the factors: confining pressure, initial dry density, initial water content, shearing rate and drainage condition to the characteristics of stress-strain of soil.

2. The inherent unity of diversity of shearing strength for the same samples measured by different experimental methods is indicated according to the normalization of critical state test results. And the failure lines in $p' - q - v$ space of remoulded saturated expansive soil under consolidated drained and undrained condition are attained.

3. Consolidated drained tests of saturated expansive soil present strain hardening regularity. And the tests have characteristics of shear dilatation. The hyperbolic curve model can fit well the weak hardening stress-strain curve. Consolidated undrained test of sample presents strain softening regularity. The exponential curve model can fit the weak softening stress-strain curves.

Acknowledgements

This research is financially supported by National Natural Science Foundation of China (NO. 41102186); Opening fund of State Key Laboratory of Geohazard Prevention and Geoenvironment Protection (Chengdu University of Technology) (SKLGP2011K001); Opening fund of State Key Laboratory for Geomechanics and Deep Underground Engineering (China University of Mining & Technology) (SKLGDUEK 1006); Opening fund of Key Laboratory of Geotechnical and Underground Engineering (Tongji University), Ministry of Education(KLE-TJGE-B0903); the Open Research Fund of State Key Laboratory of Simulation and Regulation of Water Cycle in River Basin (China Institute of Water Resources and Hydropower Research), Grant NO: IWHR-SKL – 201217.

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