



## ON THE PHYSICALLY NON-LINEAR ANALYSIS OF CYCLIC LOADED REINFORCED CONCRETE CROSS-SECTIONS WITH MATHEMATICAL OPTIMISATION

Erich Raue<sup>1</sup>, Hans-Georg Timmler<sup>2</sup>, Robert Garke<sup>3</sup>

*Dept of Reinforced Concrete Structures, Institute of Structural Engineering, Bauhaus-Universität Weimar, Marienstr. 13, 99421 Weimar, Germany*

*E-mail: <sup>1</sup>Erich.Raue@uni-weimar.de; <sup>2</sup>Hans-Georg.Timmler@uni-weimar.de;*

*<sup>3</sup>Robert.Garke@uni-weimar.de*

*Received 01 Dec 2008, accepted 03 Apr 2009*

**Abstract.** In the paper, experimental results of tension member tests are used as a basis from which to develop an extended tension stiffening model for reinforced concrete, with emphasis on the specific concrete damage and the developments of residual deformations depending on the structural loading. Two characteristics are contained in the proposed model: one describes the residual deformation behaviour along the reinforcement accounting for the cracks that cannot be closed completely, while the other describes the degradation of the concrete stiffness. Within context of non-linear analysis, the model is incorporated into an analytical approach, based on the LAGRANGE principle of minimum of total potential energy. The mechanical problem is solved with the application of the mathematical optimisation, using energy principles formulated as a kinematic formulation and transforming them into non-linear optimisation problems. It was demonstrated, that this approach is eminently suitable for analysing pre-damaged or pre-loaded reinforced concrete cross-sections under cyclic loading.

**Keywords:** tension member test, degradation of concrete stiffness, residual deformations, cross-section analysis, mathematical optimisation.

### 1. Introduction

For the computational analysis of reinforced concrete structures, subjected to cyclic loading, numerous models have been proposed in the last years. The models can be broadly distinguished on the macroscopic level in 3 types of constitutive theory: elasticity, plasticity and continuum damage theory. Also, some coupled models based on the plasticity and continuum damage theory have been recently developed. It has been proved that the models of the plasticity and continuum damage theory can accurately simulate inelastic behaviour during cyclic loading, but its practical application is reduced, because of the great number of parameters usually needed.

Many models published in the literature have been focused on particular aspects of cyclic loading. Most of them are only oriented to the compressive regime and only a few consider the cyclic tension response. Although cracking is an important aspect of concrete behaviour that considerably affects the response of reinforced concrete structures, the post-cracking resistance of concrete and its influence on the inelastic behaviour is often neglected. It has also been proved in cyclic loaded tests under serviceability conditions that the structural response, such as the reduction of stiffness and the residual deflections, are especially influenced by the inelastic behaviour in the effective area of concrete in tension.

Concrete under tension regime exhibits a gradual postpeak softening behaviour similar to the response of

concrete under uniaxial compression. Available information on the concrete subjected to cyclic tensile loading were stated in experimental and theoretical investigations of Gopalaratnam and Shah (1985), Yankelevsky and Reinhardt (1989) and Sima *et al.* (2008).

The tension stiffening stress-strain relations of concrete in cracked *r/c* structures are different from those of tension softening in plain concrete, due to the action of bond with the reinforcement. This is stated in a number of works Goto (1971), Kaklauskas and Ghaboussi (2001), Khalfallah (2008). Tension stiffening under cyclic loading is stated in (Yankelevsky *et al.* 2008; Bischoff 2003; Palermo and Vecchio 2003).

In this paper a simplified concrete tension stiffening model for the simulation of cyclic loading, including partial unloading-reloading is proposed. The model considers the increase of inelastic strain and the degradation of the concrete stiffness. The maximum deformation of the loading is assumed to be a principal parameter to determine the unloading-reloading curve and it gives the point on the envelope from which unloading starts.

The implementation of the model in an analytical approach (Raue 2007) for non-linear computation, based on the LAGRANGE principle of minimum total potential energy, is presented. In this approach the mechanical problem is solved with the aid of the mathematical optimisation, using energy principles formulated as a kinema-

tic formulation and transforming them into non-linear optimisation problems.

As an ongoing development, the presented model has been extended in an approach suitable for the non-linear analysis of entire structural elements. The implementation and the results will be published in a future paper.

2. Tension member tests

2.1. Experimental investigation

In the case of cyclic tension, results of conventional tension member tests performed by Heidolf (2007) were used in order to determine the evolution of the damage behaviour and its influence on cyclic loading. Polymer cement concrete (PCC) was used for the specimens. This is motivated by the deceleration of the tensile damage behaviour in concrete that usually provides a more detailed investigation of material changes during an increasingly cyclic loading. In the context of this paper only the analytical modelling, which is derived from the experimental results, is treated, while the mixtures of the PCC, such as the type and quantity of the polymers, are untreated.

The specimens were reinforced concrete cylinders with a diameter of 100 mm, a height of 1.000 mm and with a 12 mm diameter reinforcing bar BSt 500 embedded at the centre. Fig. 1a shows the experimental setup and the load history. The uniaxial tensile force was applied to the 2 protruding ends of the steel bar embedded in the concrete prism. The loading was carried out in incremental load steps with repetitions in order to investigate the incremental evolution of the damage. The tests were finished before the yielding of the reinforcement.

Inductive bridge displacements were attached at different locations along the tension member to investigate the longitudinal deformations. In addition to the measurement of the longitudinal deformations, the tests were also used to investigate the opening/closing behaviour of a discrete crack. Therefore additional displacement transducers were placed above the formation of the initial crack. For initiation the first crack, the specimens were

weakened prior to loading at half-height by a circular saw cut. In order to derive the material behaviour in the macroscopic level, the following assumptions are applied:

- The analysis is based on a smeared crack approach, which accounts for the deformation due to the crack opening, as well as the strain of the solid material between the cracks.
- The average strains are obtained by the longitudinal deformations related to the measurement length  $l_{meas}$

$$\epsilon_m = \epsilon_{sm} = \epsilon_{cm} = \Delta l / l_{meas} .$$

Fig. 1b shows the experimental load-deformation relationship of the cyclically loaded specimen, the analytical load-deformation relation of the embedded reinforcement and the analytically derived load-deformation relationship of the concrete inclusive tension stiffening.

The results in Fig. 1b show that cracking appears immediately with a quick decrease in the concrete force, but the tension stiffening effect, measured as the difference between the experimental curve of the tension member and the analytical curve of the rebar, is far from negligible and increases with the force applied.

In addition to the overall concrete deformation, the corresponding elastic and inelastic strain parts are of interest. The amount of inelastic strains can be attributable to the permanent deformation, when cracks are prevented to close upon unloading. Through the crack process, fine particles of concrete can be loosened and shifted in the crack surfaces, so that a residual crack opening can be observed. The elastic resilience of the deformations and the slip between concrete and reinforcement bar is confined by the clogging of the crack spacing by intrusion of fine materials. The inhibited unloading process causes a redistribution of the tensile forces from the reinforcement to the concrete. Finally, in the cross-section a self-equilibrating stress state is remained. The corresponding strains and the residual crack opening are kinematically compatible.

Fig. 2a shows the stress-strain relation of the concrete. The stepped response can be attributed to the cyclic loading mode with 5 repetitions of each load step. The average envelope curve shows a constant amount of ten-

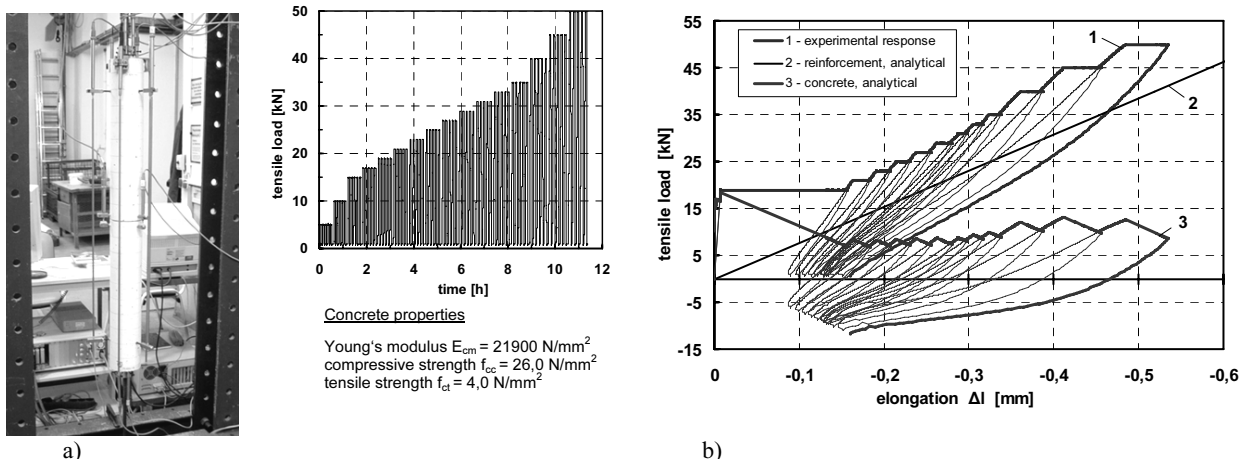


Fig. 1. Experimental analysis of tension member specimen: a – configuration of the experimental setup, load history and mechanical properties measured on the cylindrical concrete samples, b – load-deformation relationship ( $l_{meas} = 290 \text{ mm}$ ) (1. loading and 5. unloading branch of a load step)

sion stiffening  $\sigma_{ct} / f_{ct} = 0.4$  to  $0.5$ , which is in accordance to the smeared crack approach and independent of the strain.

The important issue of interest here is the development of proper tension stiffening parameters for cyclic loading. Therefore the main characteristics, i.e. the development of inelastic deformations and the degradation of the stiffness, in the following are derived. It has been observed that these characteristics vary with the accumulation of the damage.

**2.2. Inelastic deformation**

For the infinitesimal strains one can assume the additive split of the total strain  $\epsilon_{tot}$  into an elastic part and an inelastic part, i.e.  $\epsilon_{tot} = \epsilon_{el} + \epsilon_{inel}$ . Then the inelastic strain - total strain ratio can be formulated:

$$\alpha = \frac{\epsilon_{inel}}{\epsilon_{tot}}, \tag{1}$$

where the quotient of the strains is designated parameter  $\alpha$ . In Fig. 2c a strong linear correlation between  $\epsilon_{inel}$  and  $\epsilon_{tot}$  is observed. The parameter  $\alpha$  is obviously independent of  $\epsilon_{tot}$  and can be assumed constant. It follows from this assumption that the corresponding strain parts may be simply represented as linear relation of  $\alpha$  to  $\epsilon_{tot}$ :

$$\begin{aligned} \epsilon_{inel} &= \alpha \cdot \epsilon_{tot}, \\ \epsilon_{el} &= (1 - \alpha) \cdot \epsilon_{tot}. \end{aligned} \tag{2}$$

**2.3. Stiffness degradation**

The gradual degradation of the concrete stiffness can be obtained by analysing the secant modulus of the unloading and reloading curves. Herein the linear slopes of the concrete stress-strain relationships under tension are derived (Fig. 2b). It can be noted, that the decrease of the initial concrete stiffness is determined up to 85%. This is shown in Fig. 2d for the unloading and reloading path.

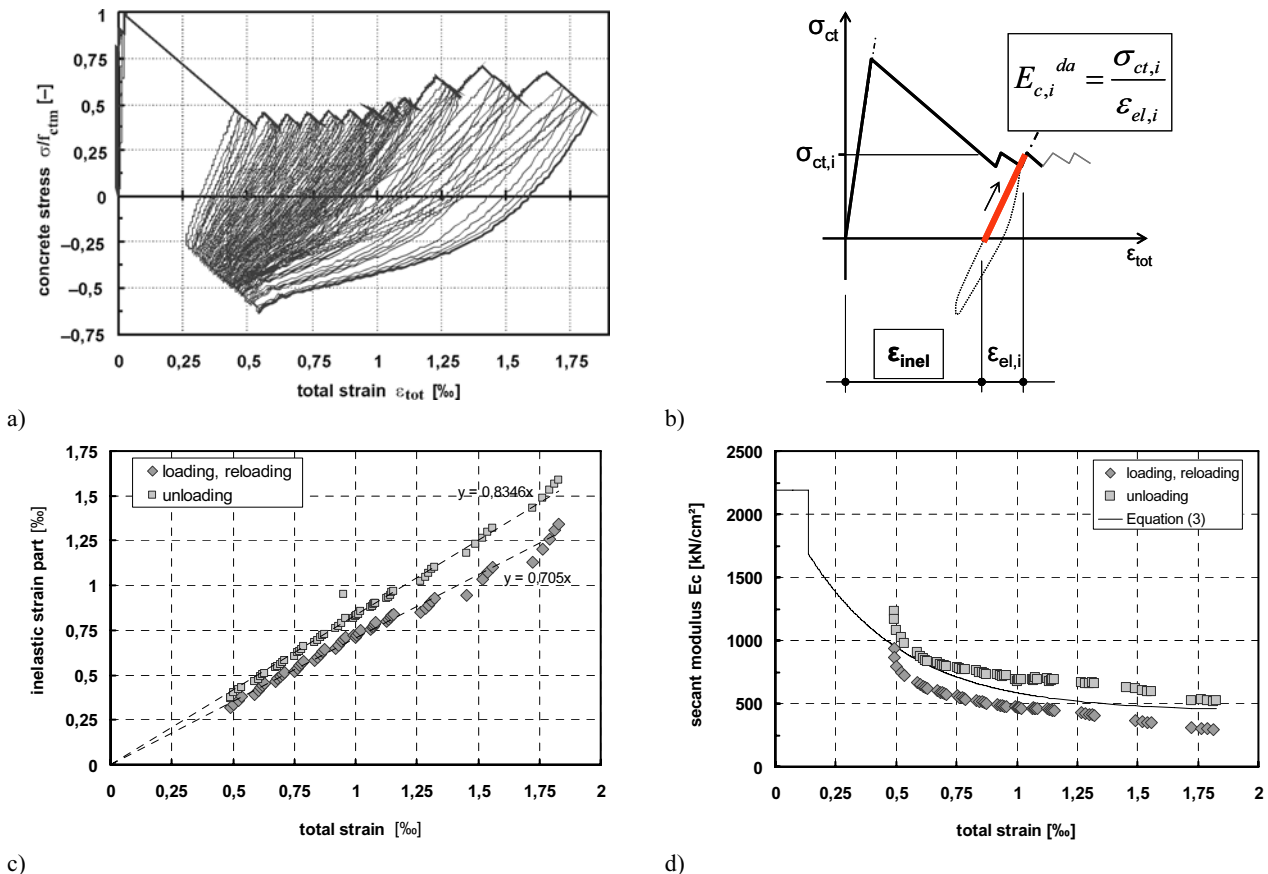
The development of the effective concrete stiffness  $E_c^{da}$  can be modeled through an adequate adjustment of the initial material stiffness  $E_c^0$  and a scalar damage variable  $D$ :

$$E_c^{da} = (1 - D(\epsilon_{inel})) \cdot E_{c0}, \tag{3}$$

where  $D$  ascertains the material degradation in tension and varies from ( $D = 0$ ) material without deterioration to ( $D = 1$ ) completely damaged material. The damage evolution is controlled by the maximum inelastic strain included in an exponential equation. Then the following relation can be utilized

$$D(\epsilon_{inel}) = a \cdot \left( 1 - e^{-(b \cdot \epsilon_{inel})^c} \right), \tag{4}$$

where  $a$ ,  $b$  and  $c$  are material parameters which should be identified by experimental results.



**Fig. 2.** Experimental results: a – concrete stress-strain relationship, b – schematic description for analysing damage parameters, c – correlation between inelastic and total strains, d – degradation of the concrete secant stiffness

### 2.4. Influence of an arbitrary load history

In case of an arbitrary or cyclic loading, the strains do not increase gradually, as considered above. In general, the inelastic strain and damage variable are related to the maximum strain of the entire load history.

In Fig. 3 an arbitrary load sequence is shown. It is assumed that the first load step causes inelastic strains  $\epsilon_{inel}$ . In the second load step the intensity of the load is reduced. Owing to the unloading  $\epsilon_{tot}$  are diminished, but the inelastic strains do not revised. Obviously, the inelastic strains are not affected by the load reduction.

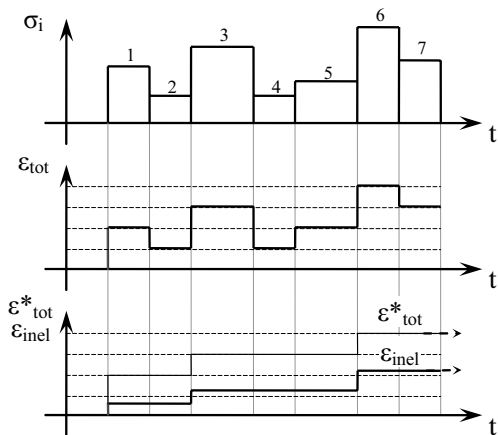


Fig. 3. Arbitrary load sequence and corresponding strain distributions

In the next step the load is higher than in the first, hence inelastic strains are naturally increased. In step 4 the load is equal to the second step, but the inelastic strains are higher than in the second step.

It is obvious, that the inelastic strain  $\epsilon_{inel,j}$  corresponding to the actual load level  $j$  is related to the total strain  $\epsilon_{tot,j}$  only when  $\epsilon_{tot,j}$  is higher than the maximum of all previous total strains  $\epsilon_{tot,i}$  with  $i < j$ . In other words, the inelastic strain remains unchanged unless the previous maximum strain in the load history is exceeded. For consideration of cyclic loading, the relation in Eq. 2 has to be extended:

$$\epsilon_{inel,j} = \begin{cases} \alpha \cdot \epsilon_{tot,j} & \epsilon_{tot,j} > \epsilon_{tot,j}^* \\ \alpha \cdot \epsilon_{tot,j}^* & \epsilon_{tot,j} \leq \epsilon_{tot,j}^* \end{cases} \quad (5)$$

where the superscript ‘\*’ denotes the relevant total strain of the load history

$$\epsilon_{tot,j}^* = \max_{i < j} \{ \epsilon_{tot,i} \}. \quad (6)$$

## 3. Non-linear analysis with mathematical optimisation

### 3.1. Analytical approach

In Raue (2007, 2004) an analytical approach for the non-linear computation of composite cross-sections using non-linear mathematical optimisation is given. The basic

features of this approach and the needs for implementing the new material models are briefly summarised in this section. The analytical approach is based on the LAGRANGE principle of Minimum Total Potential Energy  $\Pi$  with the object function:

$$\Pi = \Pi_i + \Pi_a \rightarrow \text{Minimum}. \quad (7)$$

The total potential energy  $\Pi$  represents an absolute minimum of all kinematically admissible deformation states and is determined as the sum of the strain energy  $\Pi_i$  and the potential energy of the external forces  $\Pi_a$ . Deformations are kinematically admissible if they satisfy the boundary conditions of geometrical compatibility.

From the kinematic formulation of energy principles, a non-linear optimisation problem is obtained by discretisation of the cross-section. Finally, the mechanical problem can be solved with the application of the non-linear mathematical optimisation.

In accordance with Bernoulli’s principle, the cross-section normal to the axis of the element remains plane during the deformation process, when the bond between the distinct segments parallel to the axis is assumed to be rigid. In this case the strain  $\epsilon_i(y, z)$  of an arbitrary segment  $i$  in the deformed cross-section is completely defined by 3 parameters: this could be, for example, the strain  $\epsilon_0$  at an arbitrary point and the curvatures  $\kappa_y$  and  $\kappa_z$ , related to the axis  $y$  and  $z$ :

$$\epsilon_i = \epsilon(y, z) = \epsilon_0 + \kappa_y \cdot y + \kappa_z \cdot z. \quad (8)$$

Depending on the given constitutive equation  $f_i$ , we obtain the stress  $\sigma_i$ :

$$\sigma_i = f_i(\epsilon_i) \quad (9)$$

and the integration of the stress delivers the corresponding specific strain energy  $W_i$ :

$$W_i = \int_{\epsilon_i} \sigma_i d\epsilon_i = \int_{\epsilon_i} f_i(\epsilon_i) d\epsilon_i. \quad (10)$$

The strain energy is directly included in the optimisation problem and the quantities  $\sigma_i$  and  $\epsilon_i$  are considered to be the final values of the stress and the strain. It must be pointed out, that through the definition of the stress-strain relation and their corresponding strain energy; it becomes very easy to take into account various material effects. New types or changes in the constitutive equation can be readily introduced and implemented without modification of the analytical approach.

The strain energy  $\Pi_i$  of a deformed subsegment is obtained by integration of the specific strain energy  $W_i$  across the segment area  $A_i$ :

$$\Pi_i = \int_{A_i} W_i dA_i = \int_{A_i} \int_{\epsilon_i} \sigma_i d\epsilon_i dA_i. \quad (11)$$

The strain energies of all discrete subsegments are summarised to the strain energy  $\Pi_i^Q$  of the cross-section:

$$\Pi_i^Q = \sum_{i=1}^n \Pi_{i,i}. \quad (12)$$

The potential energy of the external forces is determined by coupling the internal deformations ( $\epsilon_0, \kappa_y, \kappa_z$ ) with the internal force variables, i.e. the normal for-

ce  $N$  and the 2 bending moments  $M_y, M_z$  of the cross-section:

$$\Pi_a^Q = -(N \cdot \varepsilon_0 + M_y \cdot \kappa_y + M_z \cdot \kappa_z) \quad (13)$$

### 3.2. Proposed model for concrete in cyclic tension

Until reaching the tensile concrete strength, a linear elastic relationship is used for modelling the behaviour in tension. This is in agreement with experimental results and in common with most researchers. The followed post peak behaviour includes the mean features of damage accumulation obtained experimentally, such as the degradation of the stiffness and the inelastic strain - total strain ratio, cf. Eq. (2)–(6). Then the concrete stress is given by the following equation

$$\sigma = \begin{cases} (\varepsilon - \varepsilon_{inel}) \cdot (1 - D) \cdot E_0, & \varepsilon \geq \varepsilon_{inel} \\ (\varepsilon - \varepsilon_{inel}) \cdot \gamma_c \cdot (1 - D) \cdot E_0, & \varepsilon < \varepsilon_{inel} \end{cases} \quad (14)$$

where the degrading stiffness  $(1 - D) \cdot E_0$  and the inelastic strain  $\varepsilon_{inel}$  are explicitly related to the maximum deformation  $\varepsilon_{tot}^*$ . The relevant total strain is considered as a principal parameter to determine the unloading/ reloading curve and it gives the point on the envelope from which unloading starts. Instead of using curvilinear stress-strain relationship the unloading and the reloading curves in the model coincide and there is no energy dissipation during a cycle. For considering the transition between opening and closing of cracks, a reduction factor  $\gamma_c$  for the stiffness under compression is introduced. In Fig. 4 the verification of the model is presented. The analytical stress-strain relation is obtained for the cases  $\alpha = 0.77, a = 0.8, b = 3500, c = 1.01$  and  $\gamma_c = 0.4$ .

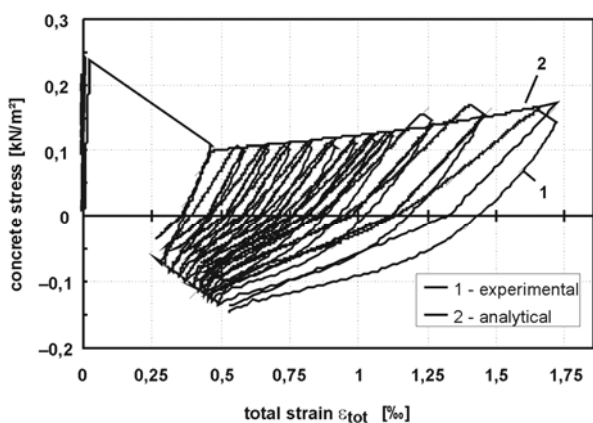


Fig. 4. Verification of the proposed model

The Eq. (14) can be rewritten in terms of the corresponding specific strain energy for unloading and reloading with strains  $\varepsilon \leq \varepsilon_{tot}^*$

$$W_i = \begin{cases} \frac{(\varepsilon - \varepsilon_{inel})^2}{2} \cdot (1 - D) \cdot E_0 & \varepsilon \geq \varepsilon_{inel} \\ \frac{(\varepsilon - \varepsilon_{inel})^2}{2} \cdot \gamma_c \cdot (1 - D) \cdot E_0 & \varepsilon < \varepsilon_{inel} \end{cases} \quad (15)$$

and for loading with strains  $\varepsilon > \varepsilon_{tot}^*$

$$W_i = \frac{f_{ct}^2}{2 \cdot E_0} + (\varepsilon - \varepsilon_{ct})^2 \cdot (1 - D) \cdot E_0, \quad (16)$$

where  $\varepsilon_{ct}$  is the concrete strain that corresponds with the tensile strength.

### 4. Numerical example

To demonstrate the performance of the approach, we analyse a semi pre-fabricated reinforced concrete cross-section of a pi-floor with additional concrete topping under cyclic loading. The proposed model and the numerical approach were implemented into the mathematical spreadsheet program Microsoft Excel. It must be stated, that due to the low number of unknowns the implementation is very simple and that the available standard solver is an adequate tool for solving the non-linear optimisation problem.

Fig. 5a shows the dimensions and material parameters of the cross-section. The mechanical problem is given for 2 several statically effective cross-sections, preliminary loaded pi-floor under construction, i.e. while pouring the uninvolved concrete topping layer; and the completed cross-section subjected to maximum load and load relief.

The implementation is performed by means of the discretisation of the cross-section, i.e. several discrete areas for the reinforcement and finite layers for the concrete, such that the strain at the centre of each layer is assumed to be constant across the layer thickness. The strain profile represents the one deformation of the cross-section that minimizes the potential energy and satisfies the kinematically admissible constraints. The stress in each layer is evaluated by using the corresponding constitutive relationship, presented in Fig. 5c.

Fig. 5b compares 2 bending moment–curvature curves analyzed with constant tension stiffening and with the proposed tension stiffening model. The responses indicate slight differences during first loading. The proposed model only estimated residual curvatures and degrading system stiffness by unloading.

Figs 5d and 5e show diagrams of the strain distribution and the corresponding concrete stress distribution across the section height. The designated “virtual” inelastic strains in the concrete top layer are assigned to the construction process and take into account the pre-deformation of the subsection pi-floor which is caused by the additional dead load of the top layer (preloading). The residual inelastic strains in the effective area of concrete in tension are developed by the damage accumulation.

The distribution of the concrete stresses shows consecutively in the first load step the response of preloaded and undamaged material, in the second load step-cracking in concrete below the neutral axis and in the third load step – the redistribution of postpeak tensile stresses into compressive concrete stresses during unloading. Further investigations are required to establish the non-linear analysis of entire structural elements.

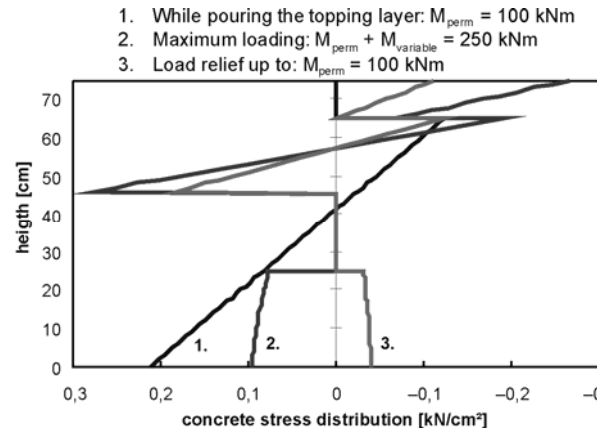
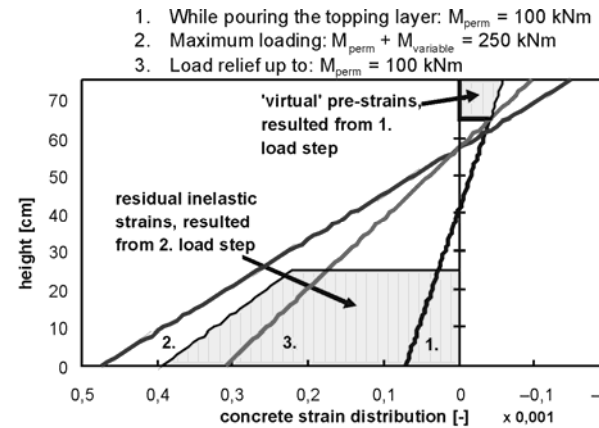
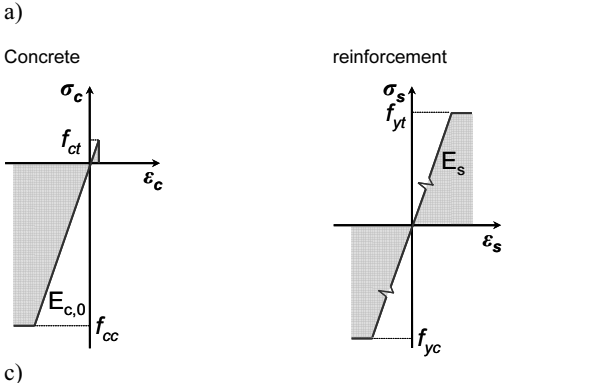
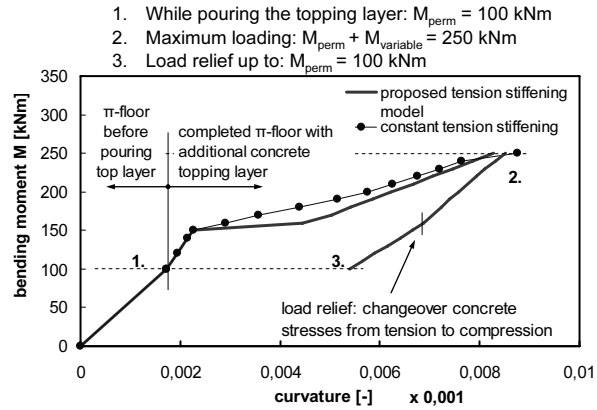
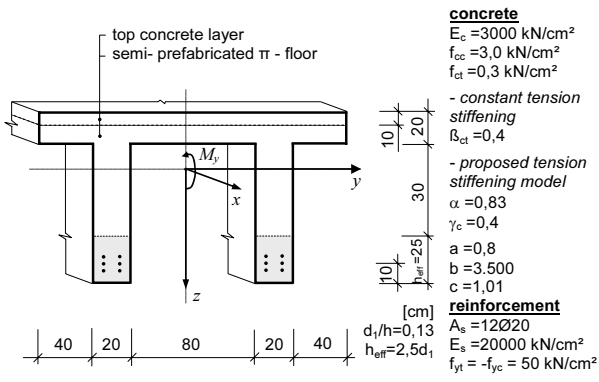


Fig. 5. Settings and results of the example. a – geometrical, material and model properties, b – relationship between bending moment and curvature, c – constitutive relationship for concrete, reinforcement and tension stiffening, d – strain distribution, e – the corresponding stress distribution above the height of the cross-section

5. Concluding remarks

Based on the investigations presented, the following conclusions can be drawn:

1. In this paper a simplified concrete tension stiffening model for cyclic loading, based on experimental results of a tension member test series, is introduced. It is capable to reproduce the concrete damage evolution in the effective area of concrete in tension. The model takes into account the stiffness degradation and the increase of inelastic deformation. Furthermore, it could be proved that the maximum deformation attained in the load history provides a criterion for describing the damage accumulation of concrete due to cyclic loading.

2. For non-linear analysis an analytical approach is used which is based on the LAGRANGE principle of the minimum total potential energy. The implementation of the model is presented by stress-strain relations and corresponding strain energies. The strain energy is directly included in the optimisation problem, hence the specifics of material behaviour can be taken into account without basic modification of the algorithm. The numerical implementation does not need special computation programme but standard spreadsheet software with options for mathematical optimisation.

## Acknowledgement

The authors gratefully acknowledged the financial support of this research from the Deutsche Forschungsgemeinschaft DFG (German Research Foundation).

## References

- Heidolf, T. 2007. *Zeit- und beanspruchungsabhängiges Tragverhalten von polymermodifizierten Beton unter mehrfach wiederholter Belastung*. PhD thesis, Weimar: Bauhaus-Universität Weimar.
- Raue, E. 2007. Non-linear analysis of composite cross-sections by non-linear optimisation, in *Proc. of the 9<sup>th</sup> International Conference Modern Building Materials, Structures and Techniques: Selected papers*, vol. 3. Ed. by M. J. Skibniewski, et al. May 16–18, 2007, Vilnius, Lithuania. Vilnius: Technika, 1040–1048.
- Raue, E. 2004. Alternative mathematical modelling in the analysis of reinforced concrete structures, in *The 8<sup>th</sup> International Conference "Modern building materials, structures and techniques: Selected papers"*, vol. 3. Ed. by M. J. Skibniewski, et al. May, 2004, Vilnius, Lithuania. Vilnius: Technika. 7 p.
- Yankelevsky, D. Z.; Jabareen, M.; Abutbul, A. D. 2008. One-dimensional analysis of tension stiffening in reinforced concrete with discrete cracks, *Engineering Structures* 30(1): 206–217.
- Khalfallah, S. 2008. Tension stiffening bond modelling of cracked flexural reinforced concrete beams, *Journal of Civil Engineering and Management* 14(2): 131–137.
- Kaklauskas, G.; Ghaboussi, J. 2001. Stress-strain relations for cracked tensile concrete from r/c beam tests, *ASCE Journal of Structural Engineering* 127(1): 64–73.
- Bischoff, P. H. 2003. Tension stiffening and cracking of steel fiber-reinforced concrete, *ASCE Journal of Materials in Civil Engineering* 15(2): 174–182.
- Palermo, D.; Vecchio, F. J. 2003. Compression field modeling of reinforced concrete subjected to reversed loading: formulation, *ACI Structural Journal* 100(5): 616–625.
- Sima, J. F.; Roca, P.; Molins, C. 2008. Cyclic constitutive model for concrete, *Engineering Structures* 30(3): 695–706.
- Yankelevsky, D. ; Reinhardt, H. W. 1989. Uniaxial behavior of concrete in cyclic tension, *ASCE Journal of Structural Engineering* 115(1): 166–182.
- Gopalratman, V. S.; Shah, S. P. 1985. Softening response of plain concrete in direct tension, *ACI Journal* 82(3): 310–323.
- Goto, Y. 1971. Cracks formed in concrete around deformed tension bars, *ACI Journal* 82(3): 310–323.

## APIE CIKLIŠKAI APKRAUTŲ GELŽBETONINIŲ SKERSPJŪVIŲ FIZINIO NETIESIŠKUMO ANALIZĘ TAIKANT MATEMATINĮ OPTIMIZAVIMĄ

E. Raue, H.-G. Timmler, R. Garke

### S a n t r a u k a

Pateikti tempiamųjų gelžbetoninių elementų eksperimentinių tyrimų rezultatai. Remiantis šiais rezultatais pasiūlytas supleišėjusio tempiamojo betono modelis, kuriame įvertinama betono pažaidų ir liekamųjų deformacijų didėjimo įtaka tempiamųjų gelžbetoninių elementų elgsenai. Modelis susideda iš dviejų komponentų: pirmasis aprašo išilginių liekamųjų deformacijų armatūroje kitimą, o antrasis betono įtakos elemento skerspjūvio deformacijoms mažėjimą. Netiesinė analizė atliekama taikant analizinį algoritmą, kuris pagrįstas Lagranžo potencinės energijos minimalumo principu. Mechaninė problema išspręsta taikant matematinį optimizavimą ir energinius principus. Pastarieji aprašyti kinematinėmis formulotėmis jas perkeliant į netiesinio optimizavimo uždavinio algoritmus. Parodyta, kad pasiūlytas algoritmas gerai tinka cikline apkrova veikiamiems gelžbetoniniams elementams analizuoti.

**Reikšminiai žodžiai:** tempiamų elementų bandymai, betono standumo mažėjimas, liekamosios deformacijos, skerspjūvio analizė, matematinis optimizavimas.

Professor Dr.-Ing. Habil. **Erich RAUE** has been working since 1976 at the Bauhaus-Universität Weimar. He is the leader of the Dept. of Reinforced Concrete Structures. He has participated in different standard committees. He teaches reinforced and prestressed concrete design, non-linear analysis of r/c structures and structural design. He conducted research in different domains: non-linear and time-depending behaviour of r/c structures under low-cyclic and impact loading, structural analysis based on non-linear optimisation methods, composite structures and alternative reinforcement, structural safety and reliability, shell structures. He has published about 170 papers and is a co-author of 3 monographies.

Dr.-Ing. Hans-Georg **TIMMLER** is teaching and research assistant at the Dept. of Reinforced Concrete Structures. His main research interests are the non-linear and time-dependent material behaviour of composites, retrofitting of existing r/c buildings, computational and structural analysis.

Dipl.-Ing. **Robert GARKE** is a PhD student at the Dept. of Reinforced Concrete Structures. His research interests include the inelastic behaviour of composites and r/c elements under alternating loads.