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L. Chodor, R. Bijak

1. Introduction

An initially curved thin-walled rod model established in this paper is a generalization of the straight thin-walled beam model presented by authors in the paper [2].

Sensitivity analysis of systems has been presented by many authors , i.e.: [5, 1 ,7]. An interesting application of the method can be found in optimization problems [5] and reliability analysis [3].

In this paper, a direct differentiation method for determining sensitivity of thin-walled structures is studied. We explore geometrically non-linear problems which are fundamental in slender structures (e.g. engineering metal structures).

The central problem in sensitivity analysis is the determination of the implicit variations in the response fields generated by a specified design variation. In general, there are three classes of methods to solve this problem: the finite difference problem, the adjoin variable method and the direct differentiation method. Finite difference sensitivity analysis methods are simple to implement, but they can be computationally expensive and deficient in terms of accuracy and reliability [7]. For this reason, the adjoin variable and direct differentiation methods are generally preferred despite their relative complexity.

2. Computation of structure sensitivity response

In this paper, we shall investigate the response of system changes $\mathbf{a}$ due to changes in basic variables $\mathbf{b}$ (such as force, geometric and material parameters) by the use of a direct differentiation method.

Computation of the response of system sensitivity by direct differentiation with the standard incremental procedure is given by [11]:

$$\mathbf{K}_T \cdot \delta\mathbf{a} = -\Psi(\mathbf{a}_I), \qquad (1)$$

$$\mathbf{a}_{I+1} = \mathbf{a}_I + \delta\mathbf{a} \qquad (2)$$

where $\mathbf{a}, \delta\mathbf{a}$ - general displacement vector and its increment respectively, $\mathbf{K}_T = \partial\Psi / \partial\mathbf{a}$ - tangent stiffness matrix, $\delta\mathbf{a}$ - increment of system response, Ψ - residual force corresponding to the level of response system $\mathbf{a}_I$, I - iteration step inside increment N. Partial derivatives of residual forces are given by

$$\mathbf{K}_T \cdot \frac{d\mathbf{a}}{db_k} = -\frac{\partial\Psi}{\partial b_k}, \qquad (3)$$

$$-\frac{\partial\Psi}{\partial b_k} = -\left(\frac{(\partial\Psi)_N}{(\partial\mathbf{a})_{N-1}} \cdot \frac{(d\mathbf{a})_{N-1}}{db_k} + \frac{(\partial\Psi)_N}{\partial b_k}\right) \qquad (4)$$

with the right side $\partial\Psi / \partial b_k$ called pseudo-load vector. In both equations (1) and (3), tangent matrix $\mathbf{K}_T$ is the same. In geometrically non-linear problem, right side vector in formula (3) is dependent on sensitivity in previous increment step N-1 as follows [7], as shown in equation (4). Derivative $(d\mathbf{a})_{N-1} / db_k$ is known from previous step, and it is determined based on the derivative $(d\mathbf{a})_{N-2} / db_k$ in advance step. We proceed in this way until start increment.

3. Initially curved thin-walled rod model

Position vectors describing the location of an arbitrary material point (X_1, X_2, S) in the initially curved thin-walled rod in the undeformed configuration $\mathbf{R}(X_\alpha, S)$ and in configuration after deformation $\mathbf{r}(X_\alpha, S)$ (a=1,2, $(X_1, X_2) \subset \Omega$, Fig 1) are given by relations (5,6).

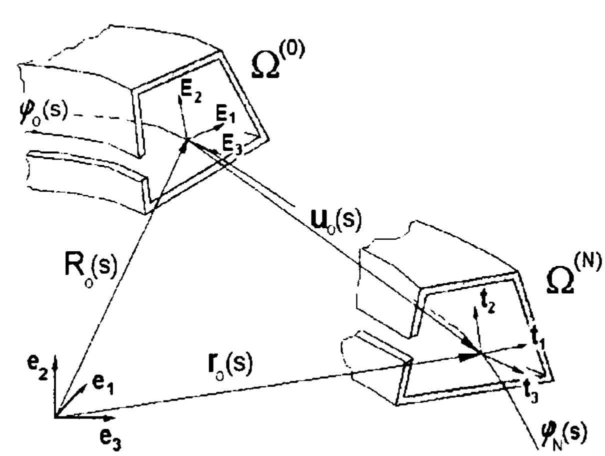

Fig 1. Kinematic description of the thin-walled rod

$$\mathbf{R}(X_\alpha,S)=\mathbf{R}_o(S)+X_\alpha\mathbf{E}_\alpha\ , \quad (5)$$

$$\mathbf{r}(X_\alpha,S)=\mathbf{r}_o(S)+X_\alpha\mathbf{t}_\alpha+f(X_1,X_2)p(S)\mathbf{t}_3 \quad (6)$$

where: $f(X_1X_2)$ - is a prescribed *(given a priori)* warping function, and $p(S)$ is the (unknown) warping amplitude. In the above equation, $\mathbf{r}_o$ describing position vector of the line of centroid and orthonormal basis $\mathbf{t}_i$ results from the rotation of the material (orthonormal) basis $\mathbf{E}_i$. Denoting the orthogonal transformation by $\Lambda=\mathbf{t}_i\otimes\mathbf{E}_i$ and inserting kinematic relation (5) into the definition of the deformation gradient tensor, the following expressions are derived:

$$\mathbf{F}=\frac{\partial\mathbf{r}}{\partial\mathbf{R}}=\frac{\partial\mathbf{r}}{\partial X_\alpha}\otimes\frac{\partial X_\alpha}{\partial\mathbf{R}}+\frac{\partial\mathbf{r}}{\partial S}\otimes\frac{\partial\mathbf{S}}{\partial\mathbf{R}}=$$
$$=\mathbf{g}_\alpha\otimes\mathbf{G}^\alpha+\mathbf{g}_3\otimes\mathbf{G}^3=\left(\mathbf{t}_a+f_{,a}p\mathbf{t}_3\right)\otimes G^a+$$
$$+\left[(\mathbf{r}_0)'+\overline{\omega}\times\left(X_a\mathbf{t}_a+fp\mathbf{t}_3\right)+fp'\mathbf{t}_3\right]$$
$$\otimes\mathbf{G}^3=\Lambda\{\mathbf{1}_3+p\mathbf{E}_3\otimes f_{,a}\mathbf{E}_a+$$
$$+\frac{1}{g_0}\omega_{03}p\left(X_2f_{,1}-X_1f_{,2}\right)\mathbf{E}_3\otimes\mathbf{E}_3+$$
$$-\frac{1}{g_0}\omega_{0i}\mathbf{E}_i\times X_a\mathbf{E}_a+\frac{1}{g_0}[\Gamma+$$
$$+\mathbf{K}\times\left(X_a\mathbf{E}_a+fp\mathbf{E}_3\right)+fp'\mathbf{E}_3]\otimes\mathbf{E}_3\} \quad (7)$$

In equation (4) $(\bullet)_{,\alpha}=\frac{\partial(\bullet)}{\partial X_\alpha}$, $(\bullet)'=\frac{\partial(\bullet)}{\partial S}$. $\mathbf{G}^i$ are contravariant base vectors in undeformed configuration [4]:

$$\mathbf{G}^1=\mathbf{E}_1+\frac{X_2\omega_{03}}{g_o}\mathbf{E}_3, \quad (8)$$

$$\mathbf{G}^2=\mathbf{E}_2-\frac{X_1\omega_{03}}{g_o}\mathbf{E}_3, \quad (9)$$

$$\mathbf{G}^3=\frac{\mathbf{E}_3}{g_o}, \quad (10)$$

$$g_o=1-X_1\omega_{02}+X_2\omega_{01}. \quad (11)$$

Centroidal line strains and curvatures are represented by vectors:

$$\Gamma=\Lambda^T\{(\mathbf{r}_o)'-\mathbf{t}_3\}=\Lambda^T(\mathbf{r}_o)'-\mathbf{E}_3, \quad (12)$$

$$\mathbf{K}=\Lambda^T\overline{\omega}=\Lambda^T\omega+\omega_0\ . \quad (13)$$

Beam curvatures are represented by the skew-symmetric tensor Ω_0, Ω, $\overline{\Omega}$ or axial vector $\omega_0,\omega,\overline{\omega}$, respectively. Curvatures before deformation are expressed by

$$(\mathbf{E}_i)'=\Omega_o\mathbf{E}_i=\boldsymbol{\omega}_o\times\mathbf{E}_i\ , \quad (14)$$

where $\Omega_o=\frac{d\Lambda_o}{dS}\Lambda_o{}^T$, (15)

and after deformation by

$$(\mathbf{t}_i)'=\left[\Omega+\Lambda\Omega_0\Lambda^T\right]\mathbf{t}_i=\overline{\Omega}\,\mathbf{t}_i=$$
$$=\left(\omega+\Lambda\omega_0\right)\times\mathbf{t}_i=\overline{\omega}\times\mathbf{t}_i \quad (16)$$

where $\overline{\Omega}=\Omega+\Lambda\Omega_o\Lambda^T$,

$$\Omega=\frac{d\Lambda}{dS}\Lambda^T. \quad (17)$$

For the application of elastic-plastic constitutive equation (actual development), it is proved convenient to introduce the second-order objective Biot strain tensor [9]:

$$\mathbf{H}=\Lambda^{\mathrm{T}}\mathbf{F}-\mathbf{1}_3=p\mathbf{E}_3\otimes f_{,a}\mathbf{E}_a+$$
$$+\frac{1}{g_0}\omega_{03}p\left(X_2f_{,1}-X_1f_{,2}\right)\mathbf{E}_3\otimes\mathbf{E}_3+$$
$$+\frac{1}{g_0}[\Gamma+(\mathbf{K}-\omega_{\mathbf{01}}\mathbf{E}_1)\times X_a\mathbf{E}_a+$$
$$+\mathbf{K}\times fp\mathbf{E}_3+fp'\mathbf{E}_3]\times\mathbf{E}_3. \quad (18)$$

Based upon the assumption of small deformation strains, but upon the arbitrary displacement and rotations, the Lagrangian strain tensor $\mathbf{E}$ is equal to corotational engineering strain tensor $\breve{\varepsilon}$:

$$\mathbf{E}=\frac{1}{2}\left(\mathbf{F}^T\cdot\mathbf{F}-\mathbf{1}_3\right)\cong\frac{1}{2}\left(\mathbf{H}+\mathbf{H}^T\right)=\breve{\varepsilon} \quad (19)$$

Invariant constitutive equation in terms of $\breve{\varepsilon}$ and its conjugate (in the sense of internal work) rotational Cauchy stress tensor $\breve{\sigma}$ can be written as:

$$\breve{\sigma}=\Lambda^{\mathrm{T}}\sigma\Lambda=\mathbf{C}:\breve{\varepsilon}\cong\Lambda^{\mathrm{T}}\mathbf{P} \quad (20)$$

where $\mathbf{P}$ represents first Piola-Kirchoff stress tensor and $\mathbf{C}$ is four-order modules tensor written in terms of the constant E and G for the isotropic elastic material.

The vectors of stress resultants: $\mathbf{n}$, N, stress couples $\mathbf{m}$, $\mathbf{M}$ in spatial and material form respectively, bi-shear M_f an bi-moment B_f are obtained by the integration of stress vector over the cross-section:

$$\mathbf{n}=\int_A\mathbf{p}^3dA\ , \quad (21)$$

$$\mathbf{N}=\Lambda^T\mathbf{n}=\int_A\breve{\sigma}_3dA, \quad (22)$$

$$\mathbf{m}=\int_A(\mathbf{r}-\mathbf{r}_0)\times\mathbf{p}^3dA\ , \quad (23)$$

$$\mathbf{M} = \Lambda^{\mathrm{T}}\mathbf{m} = \int_A \left[\Lambda^{\mathrm{T}}(\mathbf{r}-\mathbf{r}_0)\right] \times \breve{\sigma}_3 dA \tag{24}$$

$$M_f = \int_A \left[f_{,a}\breve{\sigma}_{a3} + \frac{\mathbf{1}}{g_0}\omega_{03}\left(X_2 f_{,1} - \right.\right.$$

$$\left.\left. - X_1 f_{,2}\right)\breve{\sigma}_{33}\right] dA, \tag{25}$$

$$B_f = \mathbf{E}_3 \cdot \int_A f \breve{\sigma}_3 dA \tag{26}$$

where use has been made of the relation [9]:

$$\mathbf{p}^3 = \mathbf{P} \cdot \mathbf{E}_3\ , \tag{27}$$

$$\breve{\sigma}_3 = \breve{\sigma} \cdot \mathbf{E}_3 = \Lambda^{\mathrm{T}}\mathbf{p}^3 = \breve{\sigma}_{13}\mathbf{E}_3 \tag{28}$$

Relations (21-26) in vector form are expressed by:

$$\sigma_0 = \Pi \Sigma_0 \tag{29}$$

where

$$\Sigma_0 = \left[\mathbf{N}, \mathbf{M}, M_f, B_f\right]^T_{8x1}, \tag{30}$$

$$\sigma_0 = \left[\mathbf{n}, \mathbf{m}, M_f, B_f\right]^T_{8x1}, \tag{31}$$

$$\Pi = \begin{bmatrix} \Lambda & 0 & 0 \\ 0 & \Lambda & 0 \\ 0 & 0 & \mathbf{1}_2 \end{bmatrix}_{8\times 8} \tag{32}$$

For preparation of our linearization process, we explicitly derive the linearized constitutive equations resultant from form (21-26):

$$\Delta\mathbf{N} = \int_A \mathrm{C}_3 \Delta\breve{\varepsilon}_3 dA \tag{33}$$

$$\Delta\mathbf{M} = \int_A \left[\Lambda^{\mathrm{T}}(\mathbf{r}-\mathbf{r}_0)\right] \times \left[\mathrm{C}_3 \Delta\breve{\varepsilon}_3\right] dA \tag{34}$$

$$\Delta M_f = \int_A \left[f_{,a} G \cdot \Delta\breve{\varepsilon}_{a3} + \frac{\mathbf{1}}{g_0}\left(X_2 f_{,1} - \right.\right.$$

$$\left.\left. - X_1 f_{,2}\right) E \cdot \Delta\breve{\varepsilon}_{33}\right] dA \tag{35}$$

$$\Delta B_f = \mathbf{E}_3 \cdot \int_A f \mathrm{C}_3 \Delta\breve{\varepsilon}_3 dA \tag{36}$$

where $\mathrm{C}_3 = Diag(E, G, G)$ and change strain at arbitrary point $(X_1, X_2) \subset \Omega$ is given by:

$$\Delta\breve{\varepsilon}_3 = \frac{1}{g_0}\mathbf{A_h} \cdot \Delta\varepsilon_0 . \tag{37}$$

$$[\mathbf{A}_h]_{3\times 8} = \begin{bmatrix} 0\ 0\ 1 & X_2 & -X_1 & 0 & A_1 & f \\ 1\ 0\ 0 & 0 & pf & -X_2 & A_2 & 0 \\ 0\ 1\ 0 & -pf & 0 & X_1 & A_3 & 0 \end{bmatrix} \tag{38}$$

$$A_1 = \omega_{03}\left(X_2 f_{,1} - X_1 f_{,2}\right),$$

$$A_2 = g_0 f_{,1} + K_2 f\ , \tag{39}$$

$$A_3 = g_0 f_{,2} - K_1 f\ ,$$

$$\Delta\varepsilon_0 = \left[\Delta\Gamma, \Delta\mathbf{K}, \Delta p, \Delta p'\right]^T_{8\cdot 1} \tag{40}$$

Virtual work expression (equilibrium equation) for thin-walled rod is given by:

$$\psi(\delta\mathbf{a}, \mathbf{a}) = \int_{[0,L]} \left\{\Pi^{\mathrm{T}}\mathbf{B}\delta\mathbf{a} \cdot \sigma_0 - \delta\mathbf{a} \cdot \mathbf{F}_{ext}\right\} dS =$$

$$= \psi_{int}(\delta\mathbf{a}, \mathbf{a}) - \psi_{ext}(\delta\mathbf{a}, \mathbf{a}) \tag{41}$$

where N_a, $\mathbf{B}_a$ represent the shape function and strain-displacement matrix, respectively:

$$\mathbf{B}_a^T = \begin{bmatrix} N_a' \mathbf{1}_3 & N_a[\hat{\mathbf{r}}_0']^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & N_a'\mathbf{1}_3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & N_a & N_a' \end{bmatrix}_{7\times 8} \tag{42}$$

Linearized finite element equation, is derived from (41) in form:

$$\mathbf{K}_T \Delta\mathbf{a} = \Psi_{\mathbf{int}} - \Psi_{ext} \tag{43}$$

where:

$$\mathbf{K}_T = \bigcup_e \left(\mathbf{K}^M_{ab} + \mathbf{K}^G_{ab}\right) \tag{44}$$

In equation (43), $\Delta\mathbf{a} = \left[\Delta\mathbf{r}_0, \Delta\mathbf{w}, \Delta p\right]^T_{7\times 1}$ represent the incremental degrees of freedom of centroidal position vector $\mathbf{r}_o$, orthogonal transformation tensor Λ and warping amplitude p, respectively, $\mathbf{K}^M_{ab}$ represents the material stiffness matrix, $\mathbf{K}^G_{ab}$ represents geometric stiffness matrix, $\Psi_{\mathbf{int}}, \Psi_{ext}$ represent the internal and external force vector as follows:

$$\mathbf{K}^{\mathrm{M}}_{ab} = \int_{\mathrm{L}} \mathbf{B}_a^T \cdot \Pi^T \cdot \mathbf{D} \cdot \Pi \cdot \mathbf{B}_b dS, \tag{45}$$

$$\mathbf{D} = \int_A \mathbf{A}_{\mathrm{h}}^T \cdot \mathbf{C} \cdot \mathbf{A}_{\mathbf{h}} dA, \tag{46}$$

$$\mathbf{K}^{\mathrm{G}}_{ab} = \int_{\mathrm{L}} \mathbf{L}_a^T \cdot \mathbf{b} \cdot \mathbf{L}_a dS. \tag{47}$$

In the above equation, $\mathbf{L}_a$ and $\mathbf{b}$ represent displacement gradient matrix and stress matrix, respectively:

$$\mathbf{L}_a = \begin{bmatrix} N'_a \mathbf{1}_3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & N'_a \mathbf{1}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{1}_3 & \mathbf{0} \end{bmatrix}_{9\times 7} \tag{48}$$

$$\mathbf{b} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & -\hat{\mathbf{n}} \\ \mathbf{0} & \mathbf{0} & -\frac{1}{2}\hat{\mathbf{m}} \\ \hat{\mathbf{n}} & \frac{1}{2}\hat{\mathbf{m}} & sym\{\mathbf{n}\otimes\mathbf{r}'_0 - (\mathbf{r}'_0\cdot\mathbf{n})\mathbf{1}_3\} \end{bmatrix}_{9\times 9} \tag{49}$$

4. Shape sensitivity analysis of thin-walled rod

The sensitivities of an integral expression are computed after its transformations into the isoparametric domain [10], whose shape does not depend on the design variables. The Jacobian of this transformation $\left|\mathbf{J}^h(\xi)\right|$ can be expressed in terms of the nodal coordinates, so it can also be differentiated in order to known the integral sensitivities. Using this techniques, right side pseudo-load vector ($\partial\Psi / \partial b_k$), computation of the response of system sensitivity (formulas (3-6)) is given by

$$\frac{\partial \Psi_{\mathbf{int}}(\delta\mathbf{a},\mathbf{a})}{\partial b_k} = \frac{\partial \int\limits_{[-1,1]} \Pi^T \mathbf{B}\delta\mathbf{a}\cdot\sigma_0 |\mathbf{J}| d\xi}{\partial b_k} =$$

$$= \int\limits_{[-1,1]} \left\{ \frac{\partial \Pi^T}{\partial b_k} \mathbf{B} \cdot \sigma_0 \, |\mathbf{J}| + \Pi^T \, \frac{\partial \mathbf{B}}{\partial b_k} \cdot \partial_0 \, |\mathbf{J}| + \right.$$

$$\left. + \Pi^T \, \mathbf{B} \frac{\partial \sigma_0}{\partial b_k} \cdot |\mathbf{J}| + \Pi^T \cdot \mathbf{B} \cdot \sigma_0 \cdot \frac{\partial |\mathbf{J}|}{\partial b_k} \right\} d\xi \tag{50}$$

where the sensitivity of the Jacobian is:

$$\frac{\partial |\mathbf{J}|}{\partial b_k} = |\mathbf{J}| tr\left(\mathbf{J}^{-1} \frac{\partial \mathbf{J}}{\partial b_k} \right) \tag{51}$$

We assume that cross-section consists of elements of equal thickness, location of an arbitrary point $\mathbf{X} = X_\alpha \mathbf{E}_\alpha$ of cross-section is given by isoparametric interpolation:

$$\xi,\ \eta \in [-1,1] \rightarrow X =$$

$$= \sum_{I=1}^{N^e_{node}} N_I(\xi) X_0^I + \eta \cdot \frac{t}{2} \cdot v(\xi) \tag{52}$$

where $\mathbf{X}_0^I$ is position nodal point on the middle-line in cross-section element, it is thickness of section element, $\mathrm{v}(\xi)$ is vector perpendicular to middle-line.

5. Conclusion

1. Computation sensitivity of the geometrically non-linear system response of 3-D structures consisting of thin-walled rods is effected parallely with standard incremental procedure.
2. In this paper, sensitivity for a response of system is formulated via direct differentiation method due to possibility of its simple extension to the elastic-plastic range.
3. Direct differentiation method requires analytical calculations of derivative expression of matrix occurring in residual force.
4. Isoparametric formulation make possible analysis of response sensitivity to change in structure shape.
5. The model of an initially curved thin-walled rod formulated with use of strain as a symmetric part of Biot tensor make relatively simple expressions of sensitivity analysis.
6. The adopted measure of strain leads to simple expressions of residual force, which simplifies calculations of derivatives in the method direct differentiation, whereas Green-Lagrange strain measure would lead to complicated calculations.

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PLONASIENIŲ KREIVŲ STRYPŲ JAUTRUMO ANALIZĖ

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Santrauka

Darbe siūloma nauja trimačių kreivų plonasienių konstrukcijų (arba sistemų) jautrumo analizės metodika. Laikoma, kad konstrukcijos dirba tamprioje stadijoje, tačiau gali patirti didelius poslinkius ir posūkius. Konstrukcijos jautrumas bazinių kintamųjų (pavyzdžiui, jėgų, geometrijos ar medžiagų parametrų) pokyčių atžvilgiu nustatomas tiesioginio diferencijavimo metodu, standartine Zienkiewicz ir Taylor pasiūlyta prieaugių procedūra.

Siūlomi metodai iliustruojami trimačio kreivo plonasienio strypo pavyzdžiu Strypo matematiniame modelyje įvertintos skersinės šlyties bei skerspjūvio susukimo deformacijos. Parodyta, kad siūloma metodika tinkama modeliui realizuoti baigtinių elementų metodu. Strypo baigtinis elementas formuojamas virtualaus darbo principu. Išvestos visų baigtinio elemento struktūrinių matricų išraiškos.

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