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IS IT INVERTED U? TESTING RELATIONSHIPS BETWEEN DEBT AND GROWTH: NEW EVIDENCE FROM SMOOTH TRANSITION KINK REGRESSION

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Abstract. This study examines the inverted U-shaped relationship between public debt and economic growth rates in 44 countries, covering developed, developing, and underdeveloped economies, using data from 1970 to 2020. Unlike previous studies, which often use quadratic regression, threshold, and kink regression models that may not accurately capture the true relationship, we employ the smooth transition kink regression model for this analysis. Our results show that an inverted U-shaped relationship exists in 8 out of the 44 countries studied. The turning point of this relationship, where debt levels shift from beneficial to detrimental, ranges from 23.817% to 96.674%, highlighting the variance in optimal debt levels across different countries. These findings confirm the presence of the inverted U-shaped relationship but reject the notion of a universal turning point applicable to all countries.

Keywords: debt, economic growth, inverted U, smooth transition kink regression, turning point.

JEL Classification: C22, E62, H63, O57.

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1. Introduction

Some literature suggests that an increase in public debt can have positive effects on economic growth rates, while others argue that a higher level of public debt can have negative consequences on economic growth rates (Arsić et al., 2021; Albu & Albu, 2021; Asravor et al., 2023). This discrepancy leads to conflicting findings and ambiguous conclusions (James, 1984; Elmendorf & Mankiw, 1999; Cerra & Saxena, 2008). One reason for this is that past studies often overlook non-linear effects, meaning that the relationship between public debt and economic growth rates in each country may have both similar and opposite relationships. Additionally, there may be a turning point in the relationship, leading to nonlinear effects. Specifically, while some levels of debt can support economic expansion, excessive debt can hinder growth and pose risks to financial stability (Caner et al., 2010; Baum et al., 2013; Doğan & Bilgili, 2014; Liu & Lyu, 2021; Sadiq et al., 2023).

In addition to the non-linear relationship, the identification of turning points has garnered significant attention among academics and policymakers in recent years, as it allows policymakers to determine the optimal level of public debt conducive to economic growth (Yang & Su, 2018). This issue has been subject to debate and considerable efforts to find answers,

particularly since Reinhart and Rogoff (2010) published their academic article "Growth in a Time of Debt," proposing the concept of an inverted U-shape relationship between public debt and economic growth. This suggests that public debt initially has a positive effect on the economy, but beyond a certain point, it can lead to a decrease in economic growth. Moreover, it has been found that if the public debt of both developing and developed countries exceeds 60% and 90% of GDP respectively, it significantly hampers economic growth rates. These study findings have led to a paradigm shift among policymakers in many countries, with support for fiscal contraction policies when public debt levels approach these thresholds.

In recent years, there have been studies to test the inverted U-shape relationship proposed by Reinhart and Rogoff (2010) (such as Hansen (2017), Yang and Su (2018), Ueshina and Nakamura (2019)). However, the research findings of Reinhart and Rogoff (2010) have been heavily criticized by Herndon et al. (2014), who found conflicting results. They identified errors in calculation, data selection, as well as non-standard statistical methods that affected the significance of their findings. Specifically, they discovered that both mean and median GDP growth rates do not significantly differ when public debt levels exceed 90% of GDP compared to when the public debt/GDP ratios are lower.

Additionally, the use of predefined thresholds of 30, 60, and 90 for turning points is also considered unreliable as there are no credible reasons for selecting these specific numbers. Consequently, Hansen (2017) attempted to study the turning point of public debt on the economy of the United States using the kink regression model with an unknown threshold (turning point). They found that when the level of public debt exceeds 43.8%, increasing public debt negatively affects the economy, which aligns with findings by Égert (2015) and Law et al. (2021), who also found significant deviations from the work of Reinhart and Rogoff (2010) and observed that the inverted U-shape relationship did not occur in many other countries worldwide.

In this study, we aim to enhance the existing literature by re-examining the presence of the inverted U-shaped relationship between public debt and economic growth rates. Our research spans 44 countries, including developed, developing, and low-income economies, with the objective of generating clearer and more reliable insights. We examined these countries individually over an extended period, instead of drawing conclusions based on the results of a single country, as different countries are likely to exhibit their own distinct patterns of the debt-growth nexus.

Furthermore, even though past studies may have successfully demonstrated the existence of an Inverted-U-shaped relationship between Debt and growth and identified turning points, they often relied on predetermined thresholds or preconceived assumptions (Herndon et al., 2014; Reinhart & Rogoff, 2010), which may not accurately reflect reality. This is because these turning points vary across different time periods studied and differ from country to country (Bentour, 2021). Despite attempts to address this issue using non-linear models such as Threshold regression (Hansen, 1999) and kink regression (Hansen, 2017), which allow for the estimation of the turning point as a parameter, both models assume that the slope of the regression line changes abruptly at a certain point (the kink point) (Liu et al., 2022). Consequently, they may not provide straightforward interpretations for explaining the inverted U-shaped relationship. It's crucial to recognize that both kink and threshold regression models

are more likely to capture a V-shaped relationship rather than an inverted U-shaped one.

Theoretically, the nonlinear relationship between debt and economic growth should be viewed as a gradual transition rather than a sudden switch once a certain threshold is crossed (Reinhart & Rogoff, 2010). Economic systems are intricate and dynamic, and changes in debt levels typically lead to a gradual evolution in growth patterns. Time lags in policy effects, feedback mechanisms within the economy, market expectations, and policy responses all contribute to this gradual transition (North, 2018). For instance, as debt levels rise, it may take time for the full impact on economic growth to materialize, as policies and market responses unfold gradually. Additionally, policymakers often implement adjustments over time in response to changing economic conditions, such as enacting measures to reduce deficits or stimulate growth (Swamy, 2020).

Therefore, to address the limitations of previous models used to test the relationship between public debt and economic growth rates, this study proposes retesting the relationship of both variables using the smooth transition kink regression (STKR) model introduced by Maneejuk et al. (2021). We believe that this approach will yield clearer and more accurate research outcomes compared to past studies that utilized less suitable tools. This model examines the relationship between variables using a transition function that smoothly shifts from one regression equation to another as the predictor variable changes. Typically employing a logistic function, this transition function allows for gradual shifts in the relationship rather than abrupt changes. By enabling smooth transitions, STKR can offer a more realistic depiction of the inverted U-shaped relationship compared to models assuming linear, quadratic, or sudden switch relationships. To the best of our knowledge, this study represents the first attempt to test the inverted U-shaped relationship between debt and economic growth using the STKR model.

The subsequent sections of this research include Section 2: literature review, Section 3: data and methodology, Section 4: study findings, and Section 5: conclusions and policy implications.

2. Literature review

The studies conducted on the topic of public debt and economic growth have been extensive and comprehensive. Researchers have approached this issue from various angles, seeking to understand the intricate relationship between these two critical factors. They have divided their investigations into two main categories: aggregate country-level studies and individual country-level studies.

The aggregated country-level studies, such as those by Reinhart and Rogoff (2010), unveiled a non-linear relationship between economic growth rates and public debt levels as categorized into four groups: countries with debt-to-GDP ratios ranging from 0–30%, 30–60%, 60–90%, and exceeding 90% of GDP. Upon calculating the mean and median GDP growth rates for each group, they observed significantly higher rates in the first three groups compared to the fourth. This led to the conclusion that when public debt levels exceeded 90% of GDP, there was a substantial and statistically significant decrease in economic growth rates, evident across both developed and developing nations. The discovery of this non-linear re-

lationship sparked further investigation, culminating in the establishment of a threshold for public debt not exceeding 90% of GDP. This pivotal threshold was supported by the work of Kumar and Woo (2010), who demonstrated that exceeding 90% of GDP in public debt, with each 10% increase per GDP, led to a reduction of 0.2% in the GDP growth rate.

Subsequent studies, such as Caner et al. (2010) have reinforced the idea that there is an inverted U-shaped relationship between public debt and economic growth. The study found that there were threshold points for developed and developing countries at 77% and 64% of GDP, respectively. Beyond these thresholds, an increase in public debt led to a negative impact on economic growth rates. Baum et al. (2013) identified a downward-sloping relationship between public debt and economic growth rates among 12 countries in the European Union. They found that economic growth rates were negative when public debt-to-GDP ratios ranged from 90% to 105%. Okwoche and Makanza (2023) also examined the non-linear effect of public debt on per-capita GDP growth across 24 sub-Saharan African countries spanning from 1980 to 2018, using Panel Quadratic Regression. They found an inverted U-shaped relationship between public debt and economic growth, with a debt threshold typically observed around 80–85% of GDP in most cases. These results align with the threshold of around 90% of GDP proposed by Reinhart and Rogoff (2010).

Égert (2015) utilized the panel threshold regression tool developed by Hansen (1999) and reached conclusions that deviated from those of the previously mentioned studies. Through formal econometric testing using a modified version of the Reinhart–Rogoff dataset, he aimed to explore whether public debt exceeding 90% of GDP exerts a negative nonlinear impact on economic growth. Their analysis uncovered the formidable challenge of identifying a nonlinear relationship between the public debt-to-GDP ratio and economic growth, with detection heavily contingent upon modeling decisions and data coverage. They noted that instances resembling the nonlinear pattern proposed by Reinhart and Rogoff were rare, with the negative correlation emerging primarily at very low levels of public debt, typically falling between 20% and 60% of GDP. This is corroborated by Law et al. (2021), who conducted a study encompassing 71 developing countries from 1984 to 2015. They revealed that economic growth faced significant hurdles when public debt levels exceeded 51.65% of GDP. This finding is consistent with the conclusions drawn by Panizza and Presbitero (2013), Liu and Lyu (2021), who investigated the impact of public debt on economic growth in developed countries. However, it is important to acknowledge that divergent outcomes may arise due to differences in research studies, including the analyzed time period, temporal dimension, methodology, and model specifications. Therefore, when interpreting the results of such studies, it is essential to consider these factors to gain a more comprehensive understanding of the relationship between public debt and economic growth.

For the country-specific studies, Hansen (2017) developed a kink regression model to examine the existence of a threshold in the relationship between public debt and economic growth. Analyzing data from the United States spanning from 1791 to 2009, they found that once public debt levels exceeded 43.8% of GDP, economic growth slowed down. This finding aligns with the nonlinear relationship hypothesis proposed by Reinhart and Rogoff (2010). However, the intriguing aspect was that the identified threshold contradicted the 90% of GDP threshold suggested by Reinhart and Rogoff (2010). Subsequently, Yang and Su (2018) further refined the kink regression model by allowing the threshold point to vary over time.

Their study revealed that the threshold point in the relationship between debt-to-GDP ratio and economic growth is not fixed and can change over time. Bentour (2021) also utilized the kink regression model to analyze the relationship between public debt and economic growth in 20 developed countries individually, covering the period from 1880 to 2010. The study demonstrated that the relationship between public debt and economic growth varies over time and is dependent on the state of the economy. He confirmed the existence of an inverted V-shaped relationship between public debt and economic growth in seven out of the developed countries studied.

Upon reviewing previous studies, it is evident that the inverted U-shaped relationship can occur both at the group and country levels. However, the threshold of this relationship varies depending on the time period and the specific countries under study (Ren et al., 2022). While past studies on the inverted U-shaped relationship have received significant attention and validation through various analytical methods such as threshold regression and kink regression, there is concern about the practicality of assuming a sudden switching threshold. Testing the inverted U-shaped relationship often reveals a more curved nature than can be captured by threshold regression and kink regression. Consequently, the slope coefficient may not accurately reflect the inverted U-shaped relationship, making it difficult to ascertain the validity of modelling outcomes.

Furthermore, although quadratic regression is a model commonly used to test the inverted U-shaped relationship between debt and growth (as demonstrated by Ardagna et al. (2007), Checherita and Rother (2012), Liu and Lyu (2021)), it may not accurately capture the true shape of the relationship, resulting in potential inaccuracies in the regression results. Haans et al. (2016) have also affirmed that while a significant slope of the squared variable coefficient is necessary, it alone is insufficient to establish a quadratic relationship. Confirming the presence of an inverted U-shaped relationship requires several steps (for more details, refer to Lind & Mehlum (2010).

To address these methodological limitations, our study suggests smooth transition kink regression to capture the inverted U-shaped relationship between debt and growth across 44 countries, encompassing advanced, emerging, and low-income economies. This approach has significant implications for global economic policy and debt management strategies. By more accurately identifying the turning point where debt begins to hinder growth, our research can directly inform fiscal policy decisions, influencing government spending, taxation, and overall economic strategies. Our methodology enables policymakers in each country to design specific debt management strategies and make more informed fiscal policy decisions to achieve public debt sustainability (Eberhardt & Presbitero, 2015; Heimberger, 2023). Furthermore, it can serve as the foundation for early warning systems, helping to optimize public investment and guide international financial assistance in the context of fiscal rules (Beqiraj et al., 2018). This understanding of the debt-growth relationship across different stages of economic development allows for more targeted and effective policy interventions. By providing a more precise tool for economic management, our research has the potential to contribute to more sustainable growth paths across countries at various stages of development. It offers valuable insights for both national policymakers and international financial institutions in their efforts to balance economic growth with prudent debt management.

3. Data and methodology

3.1. Data used in the study

This study collects data from 44 countries, categorized into three groups: advanced economies, emerging economies, and low-income countries, as classified by the International Monetary Fund (IMF) (n.d.). The list of countries is provided in Appendix Table A1. Specifically, 37 countries were chosen from Advanced Economies and Emerging Economies based on the research of Reinhart and Rogoff (2010), while an additional 7 countries were selected from the low-income category as classified by the IMF. The list of countries selected for analysis is summarized in Table 1, utilizing data spanning from 1970 to 2020. This time frame was chosen to ensure a fair comparison among countries within the same period, with dates before 1970 removed in some advanced countries due to data availability constraints in many developing and low-income countries. We also note that the selected countries largely correspond to those studied in the seminal works of Reinhart and Rogoff (2010) and Hansen (2017). This alignment enables better comparison with previous studies, allowing us to highlight how our smooth transition kink regression method provides new insights compared to traditional methodologies.

The variables under study include the Debt to GDP ratio (DEBT), sourced from the Historical Public Debt Database (International Monetary Fund (n.d.)) as well as the real GDP per capita growth rate (GGDP), obtained from the World Development Indicators (World Bank, n.d.). Descriptive statistics of the data and unit root tests are presented in Tables 1 and 2, respectively.

According to Table 2, Japan emerges with the highest public debt to GDP ratio at 117.793%, indicating significant government debt relative to its economic output. Despite this, Japan's GDP per capita growth is relatively low at 1.840%, suggesting challenges in translating economic output into meaningful improvements in living standards. On the other hand, Australia stands out with a comparatively low public debt-to-GDP ratio of 24.889%, signaling a strong fiscal position. Its GDP per capita growth is among the highest at 1.630%, reflecting robust economic performance and potential for improving living standards. Myanmar demonstrates rapid economic expansion with a high GDP per capita growth rate of 4.815%. However, it also exhibits a substantial public debt-to-GDP ratio of 82.871%, posing potential risks to long-term economic stability if not managed effectively. Venezuela, despite having a relatively low public debt to GDP ratio of 36.442%, experiences negative GDP per capita growth at –0.154%, indicating economic contraction or stagnation likely attributed to factors such as political instability and economic mismanagement.

Prior to investigating the presence of a nonlinear relationship between debt and growth, we performed the Augmented Dickey-Fuller (ADF) test to assess the stationarity of our series. The findings are presented in Table 1, revealing compelling evidence of stationarity in our data. This suggests that our regression outcomes will not encounter the problem of spurious regression (Ren et al., 2024).

Table 1. Descriptive statistics

Country	Public debt to GDP				GDP per capita growth			
	Mean	Maximum	Minimum	Std. Dev.	Mean	Maximum	Minimum	Std. Dev.
Argentina	47.452	147.203	9.300	30.070	0.576	9.300	–11.855	5.415
Australia	24.889	60.411	9.650	9.728	1.630	5.085	–3.437	1.544
Austria	55.786	84.757	12.743	21.910	1.868	5.950	–7.124	2.280
Belgium	97.286	135.219	38.800	29.735	1.744	6.047	–6.207	2.165
Bolivia	77.484	202.133	35.327	37.535	1.103	5.647	–10.079	3.110
Cameroon	39.825	108.645	9.038	28.060	1.030	18.495	–10.690	5.325
Chile	45.169	165.500	3.879	41.669	2.356	9.347	–14.256	4.825
Colombia	31.974	68.231	12.400	12.255	1.924	6.071	–7.791	2.535
Costa Rica	48.173	110.300	19.200	24.522	2.033	6.761	–9.775	3.038
Denmark	41.165	69.200	4.300	20.127	1.490	5.659	–5.414	2.026
Ecuador	47.214	101.163	14.653	27.378	1.356	10.756	–9.159	3.414
El Salvador	47.634	109.845	10.346	25.554	0.557	5.557	–17.304	3.881
Finland	32.569	67.912	1.700	21.388	2.052	7.096	–8.513	3.100
France	52.170	118.738	15.000	29.233	1.535	5.516	–8.050	2.162
Germany	50.109	82.382	18.100	19.099	1.709	5.870	–5.455	2.166
Ghana	37.475	92.083	5.000	21.827	1.296	11.315	–14.509	4.406
Greece	90.034	205.249	18.486	56.813	1.185	9.446	–10.016	4.162
Honduras	52.252	117.973	13.457	27.699	0.911	7.173	–10.422	3.325
India	59.372	89.330	32.700	17.561	3.314	7.299	–8.165	3.320
Ireland	65.179	120.064	25.357	25.106	3.919	23.999	–6.414	4.500
Italy	98.762	161.849	38.962	29.984	1.355	6.592	–8.671	2.724
Japan	117.793	266.176	11.773	77.049	1.840	6.905	–5.681	2.441
Kenya	43.444	81.691	13.100	17.481	1.135	17.880	–7.952	3.908
Malaysia	53.584	109.000	31.303	18.076	3.739	9.115	–9.671	3.666
Mexico	43.814	78.144	17.000	13.899	1.152	6.996	–9.274	3.480
Myanmar	82.871	233.376	24.900	66.633	4.815	12.722	–9.174	4.754
Netherlands	56.590	76.773	36.969	12.342	1.643	4.833	–4.330	1.956
Nigeria	55.885	196.299	9.222	50.724	1.165	22.182	–15.450	6.221
Norway	36.810	60.398	23.584	8.712	2.096	5.754	–2.959	1.997
Peru	35.747	63.400	19.000	11.282	1.196	10.221	–14.181	5.325
Philippines	46.362	79.200	13.000	19.143	1.734	5.803	–10.782	3.622
Portugal	65.028	137.240	14.242	35.930	2.205	13.615	–8.614	3.924
Singapore	84.792	131.186	45.562	19.178	4.553	12.514	–5.462	4.107
South Africa	38.970	78.821	22.616	10.076	0.252	4.278	–7.616	2.523
Spain	49.396	123.038	7.301	29.472	1.645	6.962	–11.234	2.914
Sri Lanka	73.709	98.255	36.826	16.267	3.424	9.003	–4.079	2.448
Sweden	50.838	84.400	26.100	17.580	1.624	6.292	–5.151	2.263
Thailand	34.658	57.826	10.700	12.469	3.986	11.336	–8.742	3.754
Turkey	36.206	75.511	18.896	12.168	2.640	9.510	–7.148	3.957
UK	55.976	108.026	36.128	17.471	1.693	6.318	–10.203	2.744
US	65.080	131.177	19.450	25.975	1.647	6.312	–3.980	2.093
Uruguay	51.214	99.900	16.600	24.638	1.837	8.123	–10.854	4.412
Venezuela	36.442	232.786	4.600	47.643	0.154	16.262	–10.801	5.405
Zimbabwe	42.736	88.000	2.358	17.416	0.310	18.441	–18.491	7.623

Table 2. ADF test

Country	Public debt to GDP	GDP per capita Growth	Country	Public debt to GDP	GDP per capita Growth
Argentina	–3.208***	–5.560***	Kenya	–1.619*	–5.825***
Australia	1.915**	–6.428***	Malaysia	–2.484**	–5.375***
Austria	–1.966**	–4.197***	Mexico	–2.107**	–4.787***
Belgium	–1.967**	–5.190***	Myanmar	–1.785*	–1.918*
Bolivia	–1.981**	–1.984**	Netherlands	–1.954**	–3.866***
Cameroon	–2.312**	–1.991**	Nigeria	–1.801*	–5.609***
Chile	–4.046***	–4.772***	Norway	–3.519***	–2.784***
Colombia	2.372**	–3.249***	Peru	–3.188***	–3.768***
Costa Rica	–2.607***	–4.280***	Philippines	–1.984**	–3.309**
Denmark	–2.375**	–5.494***	Portugal	–2.172**	–4.329***
Ecuador	–1.666*	–3.632***	Singapore	–3.534***	–4.977***
El Salvador	–2.619***	–3.268***	South Africa	–2.420***	–3.654***
Finland	–1.771*	–4.088***	Spain	–2.635***	–1.941*
France	–2.114**	–3.456***	Sri Lanka	–1.852*	–4.002***
Germany	–3.584***	–5.380***	Sweden	–2.293**	–5.321***
Ghana	–2.008**	–4.707***	Thailand	–2.714***	–3.785***
Greece	–1.920*	–3.778***	Turkey	–2.680***	–6.782***
Honduras	–1.607*	–4.907***	UK	3.508***	–3.549***
India	–2.667***	–4.924***	US	–2.943***	–5.024***
Ireland	–3.238***	–4.984***	Uruguay	–2.907***	–3.495***
Italy	–2.126**	–4.168***	Venezuela	–2.913***	–5.218***
Japan	–2.132**	–4.322***	Zimbabwe	–2.606***	–4.847***

Note: The symbols *, **, and *** denote statistical significance at the significance levels of 0.01, 0.05, and 0.10, respectively.

3.2. Methodology

The tool used to examine the existence of a nonlinear relationship between public debt levels and economic growth rates, both on a country-specific and group level, involves a three-step process. Firstly, we assess the presence of turning points in the data using the Lagrange Multiplier (LM)-test across 44 individual countries. This test helps determine if there are reversal points where the relationship transitions. Secondly, among the countries exhibiting such turning points, we employ a smooth transition kink regression model to approximate the relationship. This model allows for a flexible estimation of the relationship between debt and growth, accommodating nonlinear patterns. Lastly, we identify the kink points in the relationship, which signify the thresholds where the nonlinear association between public debt and economic growth shifts. These steps enable us to comprehensively analyze the nonlinear relationship between debt and economic growth in the individual country.

3.2.1. Smooth transition kink regression (STKR)

This study utilizes the STKR model to explain the nonlinear relationship between public debt and economic growth. This relationship can be depicted by the following Equation:

$$GGDP_t = \beta_0 + \beta^{-}(1-\mathrm{F}(DEBT_{t-2},\gamma,s))DEBT_{t-1} + \beta^{+}\mathrm{F}(DEBT_{t-2},\gamma,s)DEBT_{t-1} + \theta GGDP_{t-1} + \varepsilon_t, \quad (1)$$

where $DEBT_t$ is the debt-to-GDP percentage at year t, $GGDP_t$ is the real GDP growth rate at year t, γ is the kink parameter which represents the turning point of the nonlinear relationship. We add the lagged dependent variable ($GGDP_{t-1}$) to ensure that the error term (ε_t) is approximately serially uncorrelated (Hansen, 2017), and we also consider the lagged debt-to-GDP ratio ($DEBT_{t-1}$) to avoid endogeneity problems. It is important to note that $DEBT_{t-1}$ is split into two regimes according to the probability function $\mathrm{F}(DEBT_{t-2},\gamma,s)$, where s is a smooth parameter that always has a positive value or a parameter adjusting the slope of the curve. $DEBT_{t-2}$ is the GDP growth rate at year $t-2$, which is given as a transition variable. In this study, we consider the Logistic distribution for the probability function, thus:

$$\mathrm{F}(DEBT_{t-2},\gamma,s) = \frac{1}{1+e^{-s(DEBT_{t-2}-\gamma)}}. \quad (2)$$

To validate the existence of the inverted U-shaped relationship, the slope coefficient β^{-} should be positive while β^{+} should turn negative when $DEBT_{t-2}$ is greater than γ. Note that, if $s \to \infty$, STKR reduces to linear regression. To conduct parameter estimation in the model, Least Squares (LS) estimation can be employed as follows:

$$\hat{\boldsymbol{\beta}}(\gamma,s) = \arg\min_{\boldsymbol{\beta}} \sum_{t=1}^{T} \Big(GGDP_t - \beta_0 - \beta^{-}(1-\mathrm{F}(DEBT_{t-2},\gamma,s))DEBT_{t-1} - \cdot \beta^{+}\mathrm{F}(DEBT_{t-2},\gamma,s)DEBT_{t-1} - \theta GGDP_{t-1} \Big)^2. \quad (3)$$

As the loss function in Eq. (3) is quadratic in $\hat{\boldsymbol{\beta}}$ but non-convex in γ and s. Therefore, we use a grid search to estimate the turning point in γ and smooth parameter s according to:

$$\hat{\gamma},\hat{s} = \arg\min_{\gamma,s} \min_{\beta} \hat{\boldsymbol{\beta}}(\gamma,s), \quad (4)$$

where $\hat{\boldsymbol{\beta}} = \{\beta_0,\beta^{-},\beta^{+}\}$.

3.2.2. The testing for the occurrence of a turning point using the LM-test

This study examines the inverted U-shaped relationship between public debt levels and economic growth rates across 44 countries worldwide. However, testing this relationship is challenging because of the presence of unidentified nuisance parameters under the null hypothesis. The STKR model features parameters that are not limited by the null hypothesis but are excluded from the model when the null hypothesis is true. For example, the null hypothesis does not restrict the parameters within the transition function. To address this issue, LM test introduced by Luukkonen et al. (1988) and Teräsvirta (1994) is used. This test relies on a third-order Taylor approximation of the transition function:

$$GGDP_t = \varphi_0 + \varphi_1 DEBT_{t-1} + \varphi_2 DEBT_{t-2} + \varphi_3 DEBT_{t-2}^2 + \varphi_4 DEBT_{t-2}^3 + u_t. \quad (5)$$

In the reparametrized model, the issue of identification is resolved, and the linearity can be assessed through LM statistic, which follows a standard asymptotic χ^2 distribution under the null hypothesis. This approach offers key advantages, including the avoidance of model estimation under the alternative hypothesis and the utilization of the conventional asymptotic distribution, thereby eliminating the requirement for simulation methods to evaluate the significance of test statistics (Teräsvirta, 1998). To investigate the existence of a nonlinear model, as described by Eq. (1), the null hypothesis $H_0 : \varphi_2 = \varphi_3 = \varphi_4 = 0$ compared against the alternative hypothesis $H_0 : \varphi_2 \neq \varphi_3 \neq \varphi_4 \neq 0$. The latter implies that at least one $\varphi_i \neq 0$, indicating the significance of higher-order terms and thereby suggesting the presence of a nonlinear relationship between economic growth and debt.

4. Empirical results

The study's findings are categorized into three main sub-sections. Firstly, we conducted tests to identify turning points or examine the existence of nonlinear relationships using the LM-test method. Secondly, we presented the estimated impact of public debt on economic growth rates on a country-specific basis. Finally, a robustness check was conducted to validate our results.

4.1. Kink effects test

Since the data utilized in the analysis are time series, it is imperative to initially evaluate whether they exhibit non-linear characteristics and significance before proceeding with the smooth transition kink regression model. Table 3 presents the results of kink testing for the variables concerning the nonlinear relationship between debt levels and economic growth rates.

Table 3 presents the results of the LM-test for the presence of a kink effect in the relationship between public debt and economic growth across various countries. The p-values indicate the significance of the kink effect, with lower p-values suggesting a stronger indication of a nonlinear relationship. Among the countries examined, ten countries exhibit statistically significant nonlinear relationships between public debt and economic growth. Specifically, Australia, Belgium, Denmark, Ecuador, El Salvador, Ghana, Greece, Myanmar, South Africa, and Sri Lanka demonstrate significant kink effects, as indicated by their p-values. These findings suggest that for these countries, there exists a threshold level of public debt beyond which the relationship with economic growth changes non-linearly. This highlights the importance of considering nonlinear relationships when analyzing the impact of debt on economic growth in these specific contexts.

Table 3. Results of LM-test for kink effect

Country	p-value	Country	p-value
Argentina	0.834	Kenya	0.878
Australia	0.082*	Malaysia	0.744
Austria	0.144	Mexico	0.288
Belgium	0.036**	Myanmar	0.018**
Bolivia	0.806	Netherlands	0.138
Cameroon	0.150	Nigeria	0.189
Chile	0.164	Norway	0.828
Colombia	0.140	Peru	0.816
Costa Rica	0.830	Philippines	0.514
Denmark	0.030**	Portugal	0.598
Ecuador	0.050**	Singapore	0.554
El Salvador	0.031**	South Africa	0.080*
Finland	0.916	Spain	0.430
France	0.470	Sri Lanka	0.008***
Germany	0.564	Sweden	0.330
Ghana	0.004***	Thailand	0.824
Greece	0.084*	Turkey	0.616
Honduras	0.648	United Kingdom	0.258
India	0.102	United States	0.410
Ireland	0.154	Uruguay	0.868
Italy	0.392	Venezuela	0.830
Japan	0.870	Zimbabwe	0.442

4.2. Estimation results of smooth transition kink regression

Upon a thorough examination of the data using the STKR model reported in Table 4, it is evident that ten countries show a nonlinear relationship between public debt and economic growth. Different values of β_1^- and β_1^+ are observed in these ten countries. Among them, eight countries exhibit a clear inverted U-shaped pattern, where β_1^- is positive while β_1^+ is negative. This pattern can be seen in Australia, Belgium, El Salvador, Ghana, Greece, Myanmar, South Africa, and Sri Lanka, with each country experiencing a unique threshold (kink point) at which this reversal occurs. For example, both Belgium and Greece, considered developed countries, have relatively high kink points (96.674 and 91.823 respectively), indicating a higher tolerance for public debt before economic growth begins to decline. Conversely, low-income countries like Ghana and Myanmar display lower kink points (24.590 and 82.816 respectively), suggesting a potentially more constrained ability to manage debt levels without negatively affecting economic growth. However, Denmark and Ecuador present a different situation, with their economic growth rates increasing despite higher levels of public debt. This suggests potential factors such as investment efficiency or fiscal policies that mitigate the negative impact of debt. Both Denmark and Ecuador have implemented policies aimed at promoting stability,

such as prudent fiscal management, investment in human capital and infrastructure, and efforts to diversify their economies (Jameson, 2003; Kristensen, 2015). These factors contribute to their relative stability compared to many other countries and might explain the positive impact of debt when it exceeds the turning point.

Table 4. Smooth transition kink regression

Country		β_0	β_1^-	β_1^+	Smooth	Kink	BIC
Australia	Coef	0.071**	0.137***	–0.065***	2.310	22.603	190.893
	SE	0.034	0.037	0.025		[20.547, 24.369]	
Belgium	Coef	3.694***	–0.018***	–0.031***	1.160	96.674	220.112
	SE	0.474	0.007	0.004		[94.687, 98.66]	
Denmark	Coef	2.553***	–0.214	0.329	0.013	41.344	209.521
	SE	0.310	0.376	0.236		[38.718, 43.970]	
Ecuador	Coef	5.799***	–0.123***	0.076***	15.428	53.872	253.201
	SE	0.311	0.010	0.004		[50.081, 55.663]	
El Salvador	Coef	1.043***	0.021*	–0.173***	0.122	44.101	272.203
	SE	0.394	0.011	0.036		[41.195, 47.007]	
Ghana	Coef	–9.832***	0.907***	-0.104***	0.058	24.590	300.110
	SE	0.768	0.209	0.011		[22.546, 26.634]	
Greece	Coef	–4.503***	0.701***	–0.510***	1.237	91.823	273.501
	SE	1.230	0.203	0.102		[88.256, 93.023]	
Myanmar	Coef	3.859***	1.061***	–0.717***	0.873	82.816	302.321
	SE	0.445	0.011	0.003		[79.221, 84.020]	
South Africa	Coef	–6.421***	2.642***	-2.269***	0.004	38.286	245.606
	SE	1.122	1.137	1.010		[35.928, 40.923]	
Sri Lanka	Coef	–4.092***	0.483***	–0.709***	1.314	71.274	230.892
	SE	0.502	0.190	0.208		[69.376,73.029]	

Note: The symbols *, **, and *** denote statistical significance at the significance levels of 0.01, 0.05, and 0.10, respectively. SE stands for standard error, and Coef stands for coefficient. We do not need to report the estimated parameter θ in this context to save space in the table, as it is not pertinent to assessing the inverted U-shaped relationship. [ ] denotes the 95% confidence interval.

Table 5. Estimated kink points for inverted U-shaped turning

Country	Debt to GDP (Kink Point)	Group
Australia	24.958	Developed countries
Belgium	96.674	Developed countries
El Salvador	23.817	Developing countries
Ghana	24.590	Low-income countries
Greece	91.823	Developed countries
Myanmar	82.816	low-income countries
South Africa	38.286	Developing countries
Sri Lanka	71.274	Developing countries

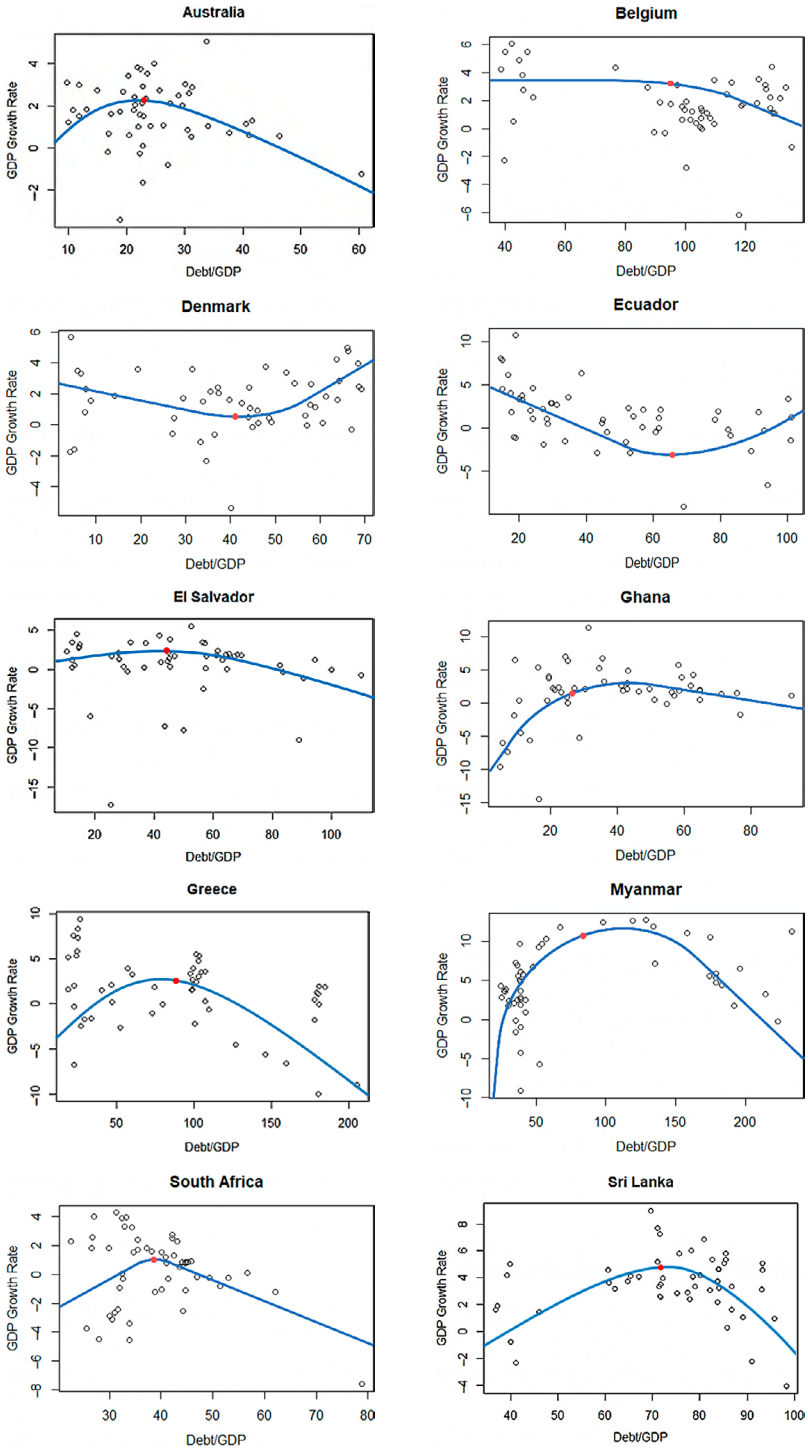

Figure 1. The non-linear relationships between public debt levels and economic growth

When comparing the levels of public debt across different country groups, it becomes apparent that the suitable debt-to-GDP criteria do not align with the classification of countries, as shown in Table 5. There are notable differences with distinct characteristics for each country, consistent with the findings of Panizza and Presbitero (2013) and Bentour (2021). These studies concluded that the turning points of the inverted U-shaped relationship between debt and growth vary among countries. Our research supports these findings, indicating that the optimal debt-to-GDP percentage is not universally applicable at 90%, as suggested by Reinhart and Rogoff (2010).

The diversity in outcomes underscores the importance of considering country-specific factors and nonlinear relationships when analyzing the impact of public debt on economic growth. This analysis provides valuable insights into identifying the optimal level of public debt for sustaining economic growth and helps illustrate the relationship between public debt and economic performance across different contexts. It facilitates informed policy decisions tailored to the specific circumstances of each country.

4.3. Robustness check

4.3.1. Comparing with the linear regression using Bayesian information criteria (BIC)

Table 6. Linear regression

Country		β_0	β_1	BIC
Australia	Coef	2.381***	–0.030	197.975
	SE	1.426	0.025	
Belgium	Coef	3.883***	–0.022***	227.213
	SE	1.008	0.010	
Denmark	Coef	0.970	0.013	226.723
	SE	0.652	0.014	
Ecuador	Coef	4.178***	–0.060***	267.448
	SE	0.851	0.016	
El Salvador	Coef	1.250	–0.015	293.365
	SE	1.165	0.022	
Ghana	Coef	–0.710	0.054**	325.439
	SE	1.203	0.028	
Greece	Coef	3.923***	–0.030***	315.506
	SE	1.011	0.010	
Myanmar	Coef	3.029***	0.022***	309.641
	SE	1.029	0.010	
South Africa	Coef	3.028***	–0.071***	255.606
	SE	1.380	0.034	
Sri Lanka	Coef	2.839**	0.008	246.683
	SE	1.620	0.021	

Note: The symbols *, **, and *** denote statistical significance at the significance levels of 0.01, 0.05, and 0.10, respectively. SE stands for standard error, and Coef stands for coefficient. We do not need to report the estimated parameter θ in this context to save space in the table, as it is not pertinent to validate our results.

The results of our analysis indicate a notable disparity between linear regression and the Structural Threshold Regression (SKTR) model in estimating the relationship between debt and economic growth. Upon re-estimation using linear regression, we observed significant changes in the regime-independent coefficients, diverging considerably from those derived from the SKTR model. Comparing the performance of both approaches using the Bayesian Information Criterion (BIC), we found that the BIC values associated with linear regression were consistently higher than those of the SKTR model (Table 6). This disparity in BIC values suggests that the SKTR model outperforms linear regression in capturing the complexities of the debt-growth relationship across the ten countries examined. It is crucial to highlight that relying solely on linear regression may introduce biases and yield unreliable results when investigating the impact of debt on economic growth. Therefore, our findings underscore the importance of employing more sophisticated nonlinear regression techniques, such as the SKTR model, for a more accurate understanding of this relationship and for informing robust policy decisions.

4.3.2. Comparing with the kink regression of Hansen (2017) and quadratic regression

To validate the results obtained from the smooth transition kink regression, we also conduct the kink regression proposed by Hansen (2017) and quadratic regression to confirm the nonlinear relationship between debt and growth in ten countries. Two statistical models can be written as follows:

$$GGDP_t = \beta_0 + \beta^{-}(DEBT_{t-1} - \gamma)_{-} + \beta^{+}(DEBT_{t-1} - \gamma)_{+} + \theta GGDP_{t-1} + \varepsilon_t; \quad (6)$$

$$GGDP_t = \beta_0 + \beta_1 DEBT_{t-1} + \beta_2 DEBT_{t-1}^2 + \theta GGDP_{t-1} + \varepsilon_t. \quad (7)$$

To test for the inverted U-shaped pattern using the results of kink regression, a similar approach to the STKR can be adopted. Specifically, the inverted U-shaped pattern is confirmed if β_1^- is positive while β_1^+ is negative. In the case of quadratic regression, confirmation of the inverted U-shaped pattern occurs when β_2 is statistically significant and negative, indicating an inverted U-shaped relationship. To identify the turning point (*TP*), we can utilize the gamma of kink regression and calculate *TP* using the Equation $TP = exp\left(-\beta_1 / \left(2\ \beta_2\right)\right)$ for quadratic regression.

According to the results presented in Tables 7 and 8, we have identified evidence of the inverted U-shaped (inverted V-shaped) relationship in Australia, Belgium, El Salvador, Ghana, Greece, Myanmar, South Africa, and Sri Lanka, thus confirming the findings of our STKR model. This robustly supports our conclusion regarding the inverted U-shaped relationship between debt and growth. Furthermore, we observe that the BIC values of both the kink regression and quadratic regression models are consistently lower than those of the STKR model for all cases. This suggests the higher potential of our models in capturing the underlying relationship between debt and growth. Additionally, it is noteworthy that some kink points or turning points obtained from quadratic regression and kink regression differ substantially from our findings. This indicates that while kink and quadratic regression models have the potential to detect the nonlinear relationship, their turning points may introduce bias.

Table 7. Quadratic regression

Country		β_0	β_1	β_2	TP	BIC
Australia	Coef	0.790	0.090***	–0.002*	22.837	1522.556
	SE	1.226	0.014	0.001		
Belgium	Coef	7.790***	0.002*	–0.130***	101.662	230.302
	SE	2.477	0.001	0.063		
Denmark	Coef	2.554***	–0.114**	0.002**	32.838	224.579
	SE	0.894	0.053	0.001		
Ecuador	Coef	7.674***	–0.231***	0.002*	74.502	265.614
	SE	1.670	0.073	0.001		
El Salvador	Coef	0.433	0.026***	–0.003***	33.890	297.004
	SE	1.947	0.008	0.001		
Ghana	Coef	–5.504***	0.370***	–0.003***	48.940	332.395
	SE	1.809	0.098	0.001		
Greece	Coef	2.334*	0.015***	–0.002*	34.661	296.617
	SE	1.322	0.038	0.001		
Myanmar	Coef	–2.702	0.191***	–0.001	129.051	305.769
	SE	1.942	0.051	0.002		
South Africa	Coef	–6.402***	0.371**	–0.004***	38.500	251.956
	SE	3.638	0.162	0.002		
Sri Lanka	Coef	–14.966***	0.590***	–0.004***	66.862	249.233
	SE	4.403	0.137	0.001		

Table 8. Kink regression

Country		β_0	β_1^-	β_1^+	Kink	BIC
Australia	Coef	1.990*	0.019**	–0.125***	31.725	203.003
	SE	1.145	0.010	0.018		
Belgium	Coef	0.853***	0.038**	–0.052***	103..495	215.061
	SE	0.118	0.019	0.013		
Denmark	Coef	2.369**	–0.045	0.087	40.182	210.311
	SE	1.292	0.067	0.081		
Ecuador	Coef	0.098	–0.164**	0.008	43.166	248.083
	SE	0.148	0.084	0.005		
El Salvador	Coef	1.041***	0.009**	–0.078***	37.048	298.508
	SE	0.361	0.004	0.011		
Ghana	Coef	–4.592***	0.358***	–0.086*	31.425	310.075
	SE	1.387	0.104	0.050		
Greece	Coef	1.992*	0.006***	–0.058*	30.102	302.231
	SE	1.201	0.002	0.026		
Myanmar	Coef	12.883***	0.166***	–0.071***	95.211	304.103
	SE	2.538	0.092	0.033		
South Africa	Coef	1.088	0.069***	–0.194*	42.266	249.211
	SE	1.326	0.016	0.113		
Sri Lanka	Coef	5.206***	0.101	–0.198**	74.954	248.324
	SE	1.245	0.078	0.105		

5. Conclusions

In this study, we question the methodology used in testing the inverted U-shaped relationship between debt and economic growth as prevalent in the literature in recent decades. Specifically, while quadratic regression has been employed to capture the inverted-shaped pattern, it may not accurately depict the true shape of the relationship, potentially leading to inaccuracies in the regression results. Moreover, a significant slope of the squared variable coefficient alone is insufficient to establish a quadratic relationship; confirming the presence of an inverted U-shaped relationship necessitates several additional steps. Furthermore, although kink regression with an unknown threshold has been proposed to estimate the turning point and simultaneously capture this nonlinear relationship, the functional form of the model is prone to fitting the V-shape rather than the U-shape. Therefore, there are limitations to both quadratic regression and kink regression in accurately characterizing the true nature of the relationship between debt and economic growth.

To this end, this study proposes a smooth transition kink regression (STKR) to re-examine the inverted-U-shaped relationship between debt and growth, aiming to provide more reliable testing results across 44 countries encompassing advanced, emerging, and low-income economies. LM-test results reveal that ten countries exhibit significant kink effects, indicating the nonlinear impact of debt on growth. Additionally, when considering the regime-dependent coefficient sign of the STKR model, we find that 8 countries display an inverted U-shaped relationship, while the other two show a U-shaped relationship. Specifically, among these, Australia, Belgium, El Salvador, Ghana, Greece, Myanmar, South Africa, and Sri Lanka demonstrate an inverted U-shaped relationship. This means that when the debt-to-GDP ratio is lower than the turning point or kink point, debt has a positive impact on economic growth, but when debt exceeds these thresholds, its impact becomes negative. In contrast, Denmark and Ecuador exhibit an opposite trend both before and after the turning point. Regarding the levels of the turning points, we observe a range from 23.817% to 96.674%, indicating that these thresholds vary across countries and are not fixed.

To validate our testing results, we employ several models, including linear and other nonlinear models such as quadratic and kink regression models. Our findings demonstrate superior performance and accuracy compared to the linear regression model. Additionally, the testing of the inverted U-shaped relationship obtained from quadratic and kink regression models also confirms our initial findings, further bolstering the robustness of our results. In addition, when comparing the performance of the STKR model with those of the quadratic and kink regression models, the Bayesian Information Criterion (BIC) results indicate the higher performance of the STKR model. This suggests that the STKR model outperforms the alternative models in capturing the nonlinear relationship between debt and economic growth across the examined countries.

Based on our findings, two key policy implications emerge for policymakers:

- First, our study reveals significant variation in debt-growth relationships across countries, highlighting the ineffectiveness of universal debt thresholds or policies. Policymakers should develop and implement debt management strategies specifically tailored to their country's economic conditions and empirically determined debt thresholds.

- Second, for the ten countries where we identify significant relationships (Australia, Belgium, Denmark, Ecuador, El Salvador, Ghana, Greece, Myanmar, South Africa, and Sri Lanka), their governments should establish systems to continuously monitor their debt-to-GDP ratios relative to their country-specific turning points. This ongoing assessment will enable policymakers to maintain growth-promoting debt levels. As debt approaches the identified thresholds, they should be prepared to enact more stringent fiscal policies or structural reforms to keep debt at productive levels.

For further investigation, it is advisable to expand the scope of the data used in the study. This research utilizes data from 1950 to 2020, encompassing 51 years, which has resulted in a limited number of observed Kink points. Therefore, it is recommended to broaden the scope of the study data, including the utilization of current datasets covering the recovery period from the second wave of COVID-19. Additionally, it is suggested to increase the sample size of countries under study, particularly focusing on the group of Low-Income countries, as there is still limited research in this area and constraints in data collection during certain periods. Furthermore, it is advisable to introduce essential fundamental variables of each country as control variables to enhance the accuracy of the data and enable a more precise analysis of the inverted U-shaped relationship. Finally, the STKR model is not specifically designed to handle outliers, which means that, like traditional regression models, its estimated coefficients may be influenced by extreme values and regress toward the mean. To better manage outliers, we suggest incorporating a quantile regression approach, which allows for estimating the model at various quantile levels and thus provides a more robust estimation by mitigating the impact of outliers.

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Author contributions

WY and PM conceived and designed the study, developed the data analysis, and wrote the first and final drafts of the article. WY prepared the methodology and estimated the model.

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APPENDIX

Table A1. List of countries

Advanced countries	Emerging countries	Low-Income countries
Australia	Argentina	Cameroon
Austria	Bolivia	Ghana
Belgium	Chile	Honduras
Denmark	Colombia	Kenya
Finland	Costa Rica	Myanmar
France	Ecuador	Nigeria
Germany	El Salvador	Zimbabwe
Greece	India	
Ireland	Malaysia	
Italy	Mexico	
Japan	Peru	
Netherlands	Philippines	
Norway	South Africa	
Portugal	Sri Lanka	
Singapore	Thailand	
Spain	Turkey	
Sweden	Uruguay	
United Kingdom	Venezuela	
United States		